

Quadratics

Skills

- ① Expand and simplify
- ② Factorise
- ③ Solve

A) Expand and Simplify

Expand means multiply. Expanding two linear factors gives four terms which may then be simplified.

Consider two lengths, $x+4$ and $x+3$.

These can be multiplied to give an area shown in the following diagram.

	x	5
x	x^2	$5x$
3	$3x$	15

$$\begin{aligned}(x+5)(x+3) &= x^2 + 5x + 3x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

This method can be applied when terms are negative.

Examples:

$$1) (x-3)(x+8)$$

$$= x^2 - 3x + 8x - 24$$

$$= x^2 + 5x - 24$$

$$2) (x-5)(x-4)$$

$$= x^2 - 5x - 4x + 20$$

$$= x^2 - 9x + 20$$

$$3) (x+1)^2 = (x+1)(x+1)$$

	x	1
x	x^2	x
1	x	1

$$= x^2 + 2x + 1$$

$$4) (2x+3)(x-4)$$

	$2x$	3
x	$2x^2$	$3x$
-4	$-8x$	-12

$$= 2x^2 - 5x - 12$$

$$5) (6-x)(3+2x)$$

	6	$-x$
3	18	$-3x$
$2x$	$12x$	$-2x^2$

$$= 18 + 9x - 2x^2$$

Factorising Quadratics

Put in brackets.

Examples

1) $x^2 + 8x + 15$

$= (x+3)(x+5)$

	x	3
x	x^2	$3x$
5	$5x$	15

2) $x^2 - 6x - 40$

$= (x-10)(x+4)$

	x	-10
x	x^2	$-10x$
4	$4x$	-40

3) $x^2 - 8x + 15$

$= (x-5)(x-3)$

	x	-5
x	x^2	$-5x$
-3	$-3x$	15

Special cases

A) No middle term (difference of two squares)

$x^2 - 49$

$= (x-7)(x+7)$

	x	-7
x	x^2	$-7x$
7	$7x$	-49

$4x^2 - 16$

	$2x$	4
$2x$	$4x^2$	
4		-16

$$x^2 - \frac{1}{9}$$

	x	$-\frac{1}{3}$
x	x^2	

$$= \left(x - \frac{1}{3}\right) \left(x + \frac{1}{3}\right) - \frac{1}{3} \quad \frac{1}{9}$$

B) Perfect Squares

$$x^2 + 10x + 25$$

	x	5
x	x^2	$5x$
5	$5x$	25

$$= (x+5)(x+5)$$

$$= (x+5)^2$$

$$x^2 - 8x + 16$$

	x	4
x	x^2	$-4x$
-4	$-4x$	16

$$= (x-4)(x-4)$$

$$= (x-4)^2$$

C) Coefficient not equal to 1

- Steps: i) look for a common factor
ii) guess and check

$$1) 3x^2 + 15x + 18$$

$$= 3(x^2 + 5x + 6)$$

$$= 3(x+3)(x+2)$$

$$3) 4x^2 - 5x - 6$$

	x	3
$4x$	$4x^2$	$12x$
-2	$-2x$	-6

X

	x	-2
$4x$	$4x^2$	$-8x$
3	$3x$	-6

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$$2) 5x^2 - 125$$

$$= 5(x^2 - 25)$$

$$= 5(x-5)(x+5)$$

$$= (x-2)(4x+3)$$

Point

check each bracket fully factorised

e.g. $(x+1)(2x+4)$ not fully factorised since $2x+4 = 2(x+2)$

$$\Rightarrow 2(x+2)(x+1)$$