

1.1 Algebra (4 credits)

Exponents

$$1. a^m \times a^n = a^{m+n}$$

$$2. \frac{a^m}{a^n} = a^{m-n}$$

$$3. (a^m)^n = a^{mn}$$

$$4. a^0 = 1 \quad (a \neq 0)$$

Examples

Simplify (1) $3a^5 \times 2a \times 4a^2 = 24a^8$

(2) $\frac{18b^8}{6b^3} = 3b^5$

(3) $\frac{15a^2b^{12}}{25a^5b^5} = \frac{3b^7}{5a^3}$

(4) $(2c^4)^5 = 32c^{20}$

Algebraic Fractions

Addition and Subtraction

We must have a common denominator

$$\begin{aligned} \text{eg. 1) } \frac{5x}{4} + \frac{x}{6} &= \frac{15x}{12} + \frac{2x}{12} \\ &= \frac{17x}{12} \end{aligned}$$

$$\begin{aligned} 2) \frac{7a}{10} - \frac{b}{4} &= \frac{14a}{20} - \frac{5b}{20} \\ &= \frac{14a - 5b}{20} \end{aligned}$$

$$\begin{aligned} 3) \frac{x-3}{5} - \frac{3x+2}{4} &= \frac{4(x-3) - 5(3x+2)}{20} \\ &= \frac{4x-12-15x-10}{20} \\ &= \frac{-11x-22}{20} \end{aligned}$$

Multiplication

$$1) \text{ Simplify } \frac{\cancel{4x^2}^x}{\cancel{18}_3} \times \frac{\cancel{10}^2}{\cancel{2}_1} = \frac{8x^2}{3}$$

$$2) \frac{\cancel{3x^3}^3}{\cancel{4}_1} \times \frac{\cancel{20y^2}^5}{\cancel{6}_2} = \frac{5x^2 y^3}{2}$$

Division

We multiply by the reciprocal

$$1) \frac{x^2}{8} \div \frac{x}{4} = \frac{x^2}{8} \times \frac{4}{x} = \frac{x^2}{2}$$

Simplify Rational Expressions

Factorise the numerator and denominator and then cancel factors.

$$\text{eg } 1) \frac{6x^2 - 18}{(x+1)(x-3)} = \frac{6(\cancel{x-3})}{(x+1)(\cancel{x-3})}$$

$$= \frac{6}{x+1}$$

$$2) \frac{a-2}{3a-6} = \frac{\cancel{a-2}}{3(\cancel{a-2})}$$

$$= \frac{1}{3}$$

Linear Equations

1. First get rid of any fractions
2. Expand brackets
3. Combine like terms
4. Get x 's on the same side
5. Solve for x

Examples

1. $5x + 7 = 10$

$$5x = 3$$

$$x = \frac{3}{5}$$

2. $\frac{3x}{5} - 4 = 5$

$$3x - 20 = 25$$

$$3x = 45$$

$$x = 15$$

3. $5x(3x - 4) - 8(x - 3) = 32$

$$15x^2 - 20x - 8x + 24 = 32$$

$$7x + 4 = 32$$

$$7x = 28$$

$$x = 4$$

4. $7x - 12 = 3x + 30$

$$4x - 12 = 30$$

$$4x = 42$$

$$x = \frac{42}{4}$$

$$x = \frac{21}{2}$$

5. $8 - 3x = 2x + 23$

$$8 = 5x + 23$$

$$-15 = 5x$$

$$x = -3$$

6. $12 - 2(3x - 1) = \frac{x + 4}{3}$

$$36 - 6(3x - 1) = x + 4$$

$$36 - 18x + 6 = x + 4$$

$$42 - 18x = x + 4$$

$$42 = 19x + 4$$

$$38 = 19x$$

$$x = 2$$

Word Problems

1. A bus company has 5 small buses and 2 large buses. A large bus can carry 14 more passengers than a small bus. In total the company can carry 224 passengers.

How many seats are there on a small bus?

Let x = number of seats on small bus

$x + 14$ = number of seats on large bus

$$5x + 2(x + 14) = 224$$

$$5x + 2x + 28 = 224$$

$$7x + 28 = 224$$

$$7x = 196$$

$$x = 28$$

There are 28 seats on a small bus

2. A washing machine takes twice as long to fill with water as it takes to empty, and the entire cycle takes 32 mins.

Fill	Wash 5min	Empty	Fill	Rinse 6min	Empty
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How long does it take to empty the machine?

Let x = time to empty

$2x$ = time to fill

$$2x + 5 + x + 2x + 6 + x = 32$$

$$6x + 11 = 32$$

$$6x = 21$$

$$x = \frac{21}{6}$$

Time to empty is 3.5 minutes

Inequalities

$<$ means "less than"

\leq means "less than or equal to"

$>$ means "greater than"

\geq means "greater than or equal to"

Examples

1) Solve

$$2x - 3 < \frac{x}{4} + 1$$

$$8x - 12 < x + 4$$

$$7x - 12 < 4$$

$$7x < 16$$

$$x < \frac{16}{7}$$

2) $x - 3(x - 2) > 10$

$$x - 3x + 6 > 10$$

$$-2x + 6 > 10$$

$$-2x > 4$$

$$x < -2$$

Note

When dividing or multiplying an inequality by a negative number, we must reverse the inequality.

3) There are 20 people on a bus trip. Adults pay \$8 and children pay \$5. To cover the cost of the bus \$125 is required in fares.

Write an inequality to describe this situation, and solve it to find the minimum number of adults required.

Let the number of adults to be x .

number of children be $20 - x$

$$8x + 5(20-x) \geq 125$$

$$8x + 100 - 5x \geq 125$$

$$3x + 100 \geq 125$$

$$3x \geq 25$$

$$x \geq 8\frac{1}{3}$$

There must be at least 9 adults on the bus.

Rational Equations

1) Solve $\frac{5x}{6} - \frac{3x}{4} = 3$

Multiply all terms by 12

$$\frac{60x}{6} - \frac{36x}{4} = 36$$

$$10x - 9x = 36$$

$$x = 36$$

2) $\frac{(x-4)}{5} - \frac{(x+3)}{10} = 2$

Multiply both sides by 10

$$\frac{10(x-4)}{5} - \frac{10(x+3)}{10} = 20$$

$$2(x-4) - (x+3) = 20$$

$$2x - 8 - x - 3 = 20$$

$$x - 11 = 20$$

$$x = 31$$

Rearranging formulae

1) Make t the subject of the formula $v = u + at$

$$v - u = at$$

$$t = \frac{v - u}{a}$$

2) Make v the subject of the formula $E = \frac{1}{2}mv^2$

$$2E = mv^2$$

$$\frac{2E}{m} = v^2$$

$$v = \sqrt{\frac{2E}{m}}$$

3) Make r the subject of $A = 4\pi r^2$

$$\frac{A}{4\pi} = r^2$$

$$r = \sqrt{\frac{A}{4\pi}}$$

4) Make x the subject of $P = 2\sqrt{x}$

$$P^2 = 4x$$

$$x = \frac{P^2}{4}$$

or

$$P = 2\sqrt{x}$$

$$\frac{P}{2} = \sqrt{x}$$

$$x = \left(\frac{P}{2}\right)^2$$

5) Make y the subject of $3x - 5y = 12$

Add $5y$ to both sides

$$3x = 12 + 5y$$

$$3x - 12 = 5y$$

$$y = \frac{3x - 12}{5}$$

Simultaneous Equations

Comparison Method

We use this method when both equations are in the form $y = mx + c$
eg Solve Simultaneously

$$\begin{aligned} 1) \quad y &= 5x - 2 \\ y &= 2x + 10 \\ 5x - 2 &= 2x + 10 \\ 3x - 2 &= 10 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} y &= 5x - 2 \\ y &= 18 \end{aligned}$$

$$\begin{aligned} \text{Solution is } x &= 4 \\ y &= 18 \end{aligned}$$

2) Solve Simultaneously

$$\begin{aligned} y &= 5x + 6 \\ y &= 10x + 8 \\ 10x + 8 &= 5x + 6 \\ 5x + 8 &= 6 \\ 5x &= -2 \\ x &= -\frac{2}{5} \end{aligned}$$

$$\begin{aligned} y &= 5x + 6 \\ y &= \left(-\frac{2}{5}\right) \end{aligned}$$

$$y = 4$$

Substitution Method

We use this method when one equation is in the form $y = mx + c$ and the other is in the form $ax + by = c$

e.g. Solve Simultaneously

$$\begin{aligned} 1) \quad y &= 4x - 3 \\ 2x + y &= 9 \end{aligned}$$

$$2x + (4x - 3) = 9$$

$$6x - 3 = 9$$

$$6x = 12$$

$$x = 2$$

$$y = 4(2) - 3$$

$$y = 5$$

$$\begin{aligned} 2) \quad 2x - 3y &= 11 \\ y &= x - 4 \end{aligned}$$

$$2x - 3(x - 4) = 11$$

$$2x - 3x + 12 = 11$$

$$-x + 12 = 11$$

$$-x = -1$$

$$x = 1$$

$$y = -3$$

The annual health costs for an adult are \$500 more than for a child. The 50 adults and 40 children of a small town have an annual health bill of \$52,000.

Write a set of equations to describe the situation above, and solve them simultaneously to find the annual health cost for an adult and for a child.

Answer:

Let cost for a adult = A

cost for child = C

$$A = C + 500$$

$$50A + 40C = 52000$$

$$50(C + 500) + 40C = 52000$$

$$50C + 25000 + 40C = 52000$$

$$90C + 25000 = 52000$$

$$90C = 27000$$

$$C = 300$$

$$A = 800$$

A child costs \$300 and an adult costs \$800

Elimination Method

We use this method when both equations are in the form $ax + by = c$.

e.g. Solve Simultaneously:

1) $3x + 2y = 19$

$$\begin{array}{r} x - 2y = 1 \\ \text{add} \quad 4x \qquad \qquad = 20 \\ \hline x \qquad \qquad = 5 \end{array}$$

$$\begin{array}{r} 15 + 2y = 19 \\ 2y \qquad \qquad = 4 \\ y \qquad \qquad = 2 \end{array}$$

2) $x + 3y = 4$

$$\begin{array}{r} 4x + 3y = 1 \\ \text{sub} \quad -3x \qquad \qquad = 3 \\ \hline x \qquad \qquad = -1 \end{array}$$

$$\begin{array}{r} -1 + 3y = 4 \\ 3y \qquad \qquad = 5 \\ y \qquad \qquad = \frac{5}{3} \end{array}$$

3) $3x - y = 37$
 $2x + 3y = 10$

$$\begin{array}{r} 9x - 3y = 111 \\ 2x + 3y = 10 \\ \text{add} \quad \hline 11x \qquad \qquad = 121 \\ x \qquad \qquad = 11 \end{array}$$

$$\begin{array}{r} 22 + 3y = 10 \\ 3y \qquad \qquad = -12 \\ y \qquad \qquad = -4 \end{array}$$

1.1 Algebra (continued)

Quadratic algebra

Expanding quadratics involves removing the brackets.

$$1) (x+a)(x+b) = x^2 + \underset{\substack{\uparrow \\ \text{add}}}{(a+b)}x + \underset{\substack{\uparrow \\ \text{multiply}}}{ab}$$

$$- (x+8)(x+5) = x^2 + 13x + 40$$

$$- (x-7)(x+4) = x^2 - 3x - 28$$

$$- (a+4)(a-4) = a^2 - 16$$

$$- (m-3)^2 = (m-3)(m-3) = m^2 - 6m + 9$$

$$- (3x-4)(x+5) = 3x^2 + 11x - 20$$

$$- (x+7)(3-x) = -x^2 - 4x + 21$$

Factorisation

1. First look for common factors
2. Next look for "difference of 2 squares"
3. Finally look for 2 linear factors.

$$\text{eg } 1) 3x^2 - 8x = x(3x-8)$$

$$2) 8t - 12t^2 = 4t(2-3t)$$

$$3) x^2 - 100 = (x+10)(x-10)$$

$$4) 9 - 25k^2 = (3+5k)(3-5k)$$

$$5) x^2 + 3x - 40 = (x+8)(x-5)$$

$$6) x^2 - 9x + 8 = (x-1)(x-8)$$

Factorise Completely

$$\begin{aligned} 1) \quad 5a^2 - 45 &= 5(a^2 - 9) \\ &= 5(a + 3)(a - 3) \end{aligned}$$

$$\begin{aligned} 2) \quad 3x^2 + 9x - 84 &= 3(x^2 + 3x - 28) \\ &= 3(x + 7)(x - 4) \end{aligned}$$

Harder Factorisation

$$1) \quad 2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

$$\begin{array}{cc} 2x & x \\ & \times \\ 1 & 3 \end{array}$$

$$2) \quad 5x^2 + 12x - 9 = (5x - 3)(x + 3)$$

$$\begin{array}{cc} 5x & x \\ & \times \\ 1 & -3 \\ 3 & \end{array}$$

$$3) \quad 6x^2 + 23x - 4 = (6x - 1)(x + 4)$$

$$\begin{array}{cc} 3x & 2x \\ & \times \\ 1 & -2 \\ 2 & \end{array}$$

$$\begin{array}{cc} 3x & 2x \\ & \times \\ 1 & -1 \\ 4 & \end{array}$$

$$\begin{array}{cc} 6x & x \\ & \times \\ -1 & 4 \end{array}$$

Simplify by first factorising

$$\textcircled{1} \frac{x^2 + 2x - 15}{2x - 6} = \frac{(x+5)(\cancel{x-3})}{2(\cancel{x-3})}$$

$$= \frac{x+5}{2}$$

$$\textcircled{2} \frac{x^2 - 16}{x^2 + 3x - 28} = \frac{(x+4)(\cancel{x-4})}{(x+7)(\cancel{x-4})}$$

$$= \frac{x+4}{x+7}$$

Quadratic Equations

1. First make R.H.S = 0
2. Factorise
3. Solve for x

Examples

$$\textcircled{1} (x+3)(x-8) = 0$$
$$x = -3 \text{ or } x = 8$$

$$\textcircled{2} x(5-x) = 0$$
$$x = 0 \text{ or } x = 5$$

$$\textcircled{3} 3(x+8)(2x-1) = 0$$
$$x = -8 \text{ or } x = \frac{1}{2}$$

$$\textcircled{4} 5x(3x+5) = 0$$
$$x = 0 \text{ or } x = -\frac{5}{3}$$

$$\textcircled{5} a^2 + 3a - 40 = 0$$
$$(a+8)(a-5) = 0$$
$$a = -8 \text{ or } a = 5$$

$$\textcircled{6} x^2 + 8 = 6x$$
$$x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
$$x = 2 \text{ or } x = 4$$

$$\textcircled{7} 28 - x^2 = 3x$$
$$0 = x^2 + 3x - 28$$
$$0 = (x+7)(x-4)$$
$$x = -7 \text{ or } x = 4$$

$$8) \quad x + 2 = \frac{323}{x}$$

$$x^2 + 2x = 323$$

$$x^2 + 2x - 323 = 0$$

$$(x + 19)(x - 17) = 0$$

$$x = -19 \quad \text{or} \quad x = 17$$

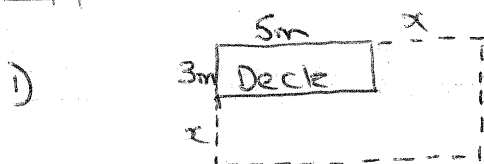
$$9) \quad (x - 5)^2 = 9$$

$$x - 5 = \pm 3$$

$$x = 5 \pm 3$$

$$x = 8 \quad \text{or} \quad x = 2$$

Applications of Quadratics



The deck measures $5\text{m} \times 3\text{m}$

It is extended by x in each direction by adding a concrete area.

The area of the concrete is 48m^2 .

Find the value of x .

$$\begin{aligned} \text{Concrete area} &= (x+5)(x+3) - (5 \times 3) \\ &= (x^2 + 8x + 15) - (15) \\ &= x^2 + 8x \end{aligned}$$

$$\text{Given} \quad x^2 + 8x = 48$$

$$x^2 + 8x - 48 = 0$$

$$(x + 12)(x - 4) = 0$$

$$x = -12 \quad \text{or} \quad x = 4$$

The value of x is 4 metres

2) The height (h) of a sky rocket above the ground after t seconds is given by $h = 50t - 5t^2$

Find the two times that it is at a height of 105m.

$$105 = 50t - 5t^2$$

$$5t^2 - 50t + 105 = 0$$

$$5(t^2 - 10t + 21) = 0$$

$$5(t - 3)(t - 7) = 0$$

$$t = 3 \text{ seconds or } t = 7 \text{ seconds}$$