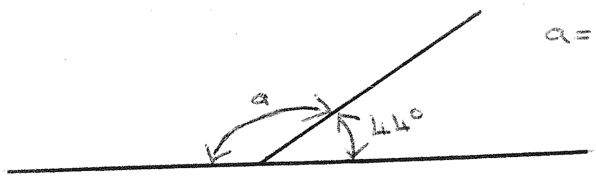
Achievement Standard 1.9

Geometric Reasoning

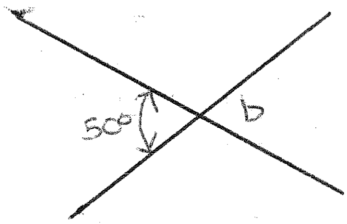
Lines and triangles

1\.



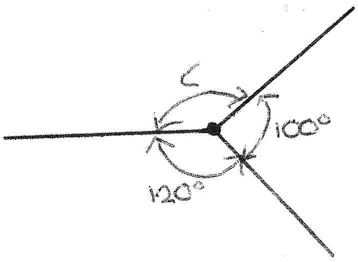
$a = 136°$ (angles on a straight lin
$\angle s$ on st. ℓ

2\.



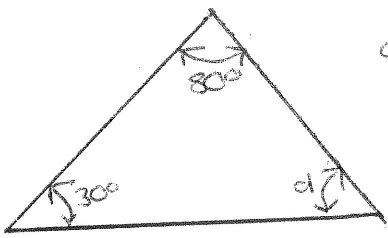
$b = 50°$ (vert. opp. $\angle s$)

3\.



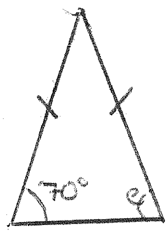
$c = 140°$ ($\angle s$ at a point)

4\.



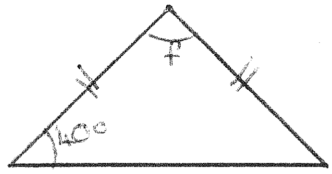
$d = 70°$ ($\angle$ sum of Δ)

5\.



$e = 70°$ (base $\angle s$, isos. Δ)

6.



$f = 100°$ (∠ sum isos. △)

7.

$g = 110°$ (ext. ∠ of △)

<u>Complementary Angles</u> - add to 90°

eg. Find the complement of 58°

Complement of 58° is 32°

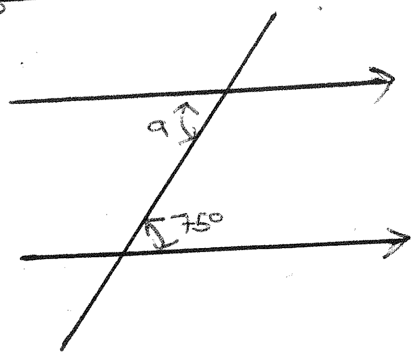
<u>Supplementary Angles</u> - add to 180°

eg. Find the supplement of 58°

Supplement of 58° is 122°

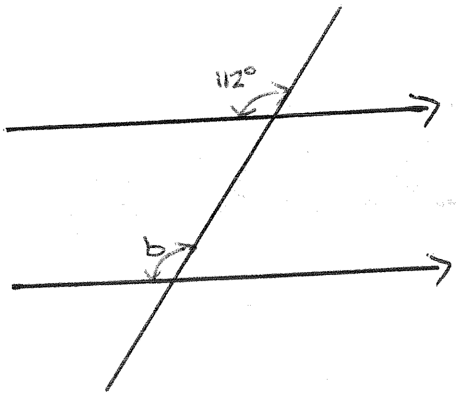
Angles and Parallel lines

Alternate Angles



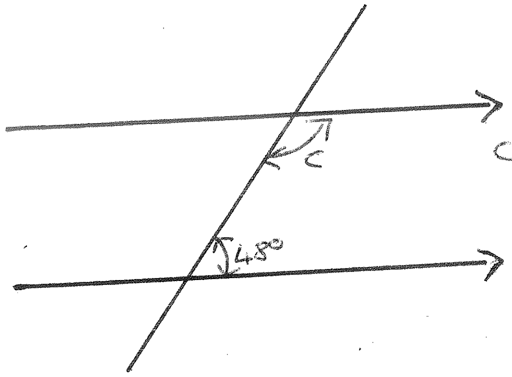
$a = 75°$ (alt. $\angle s$, // lines)

Corresponding Angles



$b = 112°$ (corresp. $\angle s$, // li

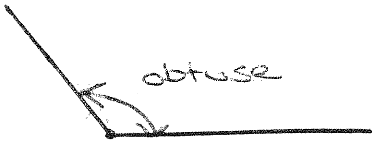
Co-interior Angles



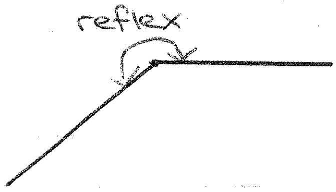
$c = 132°$ (cont. $\angle s$, // lir

Angle types

(less than 90°).



(between 90° and 180°)

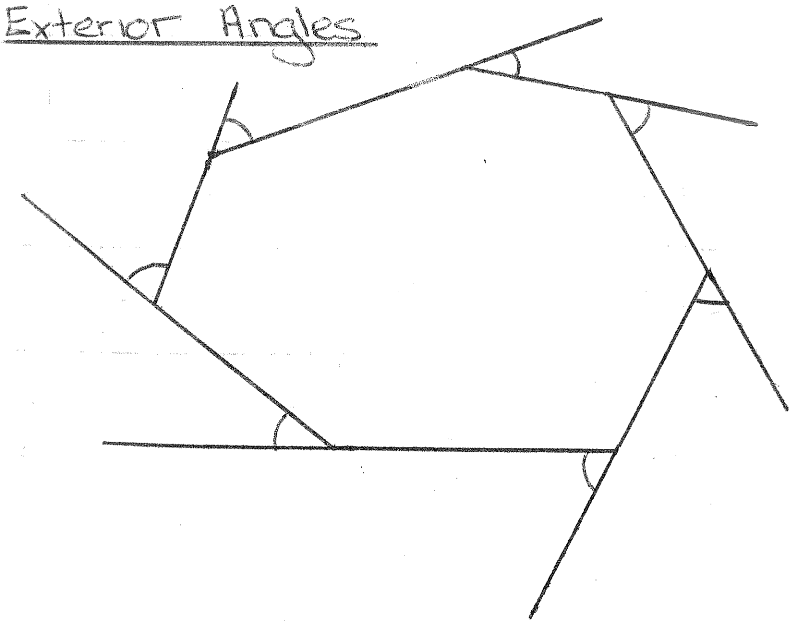


(more than 180°)

Polygons - is a many sided figure

Number sides n	Name
4	quadrilateral
5	pentagon
6	hexagon
8	octagon

Exterior Angles



For any polygon, the sum of the exterior angles is 360° .

Examples 1) Find the size of the exterior angle of a regular pentagon

$$\text{Ext.} \angle = \frac{360^\circ}{5} = 72^\circ$$

2) A regular polygon has exterior angles of 45°
How many sides does the polygon have?

$$n = \frac{360^\circ}{45^\circ} = 8$$

<u>Interior Angles</u>

The sum of the interior $\angle s$ of a n sided figure is $\underline{(n-2) \times 180^\circ}$

1) Find the sum of the interior $\angle s$ of a hexagon.

$$\begin{aligned} \text{Sum} &= (n-2) \times 180^\circ \\ &= (6-2) \times 180^\circ \\ &= 720^\circ \end{aligned}$$

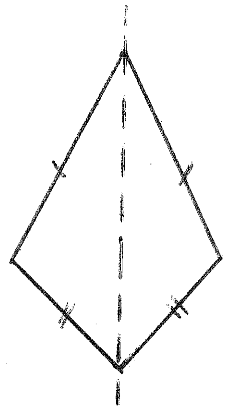
2) Find the size of the interior of a regular pe

$$\begin{aligned} \text{Sum int. } \angle s &= (5-2) \times 180^\circ \\ &= 540^\circ \end{aligned}$$

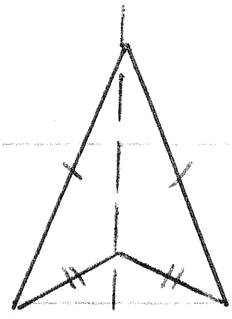
$$\text{Each interior } \angle = \frac{540}{5} = 108^\circ$$

Properties of Quadrilaterals

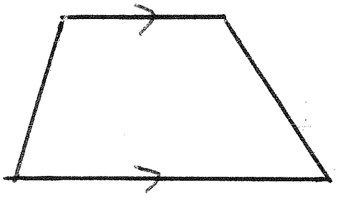
Kite



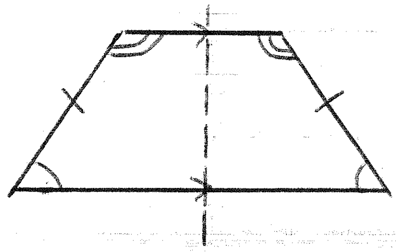
Arrow head



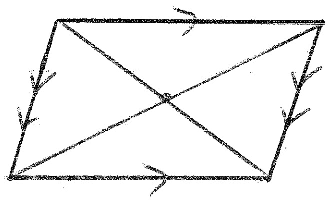
Trapezium



Isosceles Trapezium

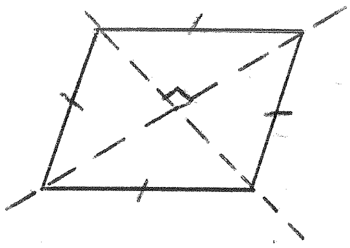


Parallelogram



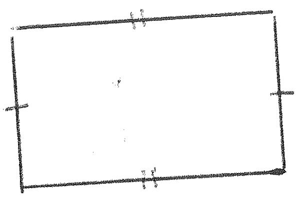
Has point symmetry about intersection of the diagonals

Rhombus



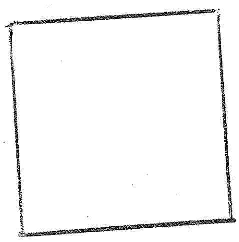
- parallelogram with 4 equal sides
- both diagonals are axes of symmetry
- diagonals bisect each other
- diagonals intersect at 90°

Rectangle



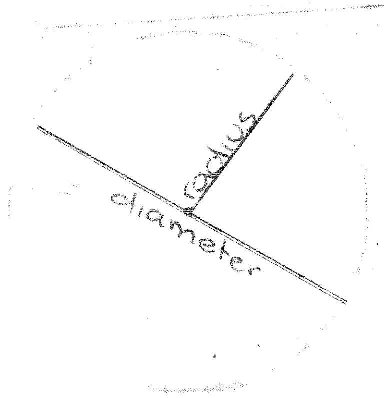
- parallelogram with 4 equal sides
- both mediators are axes of symmetry

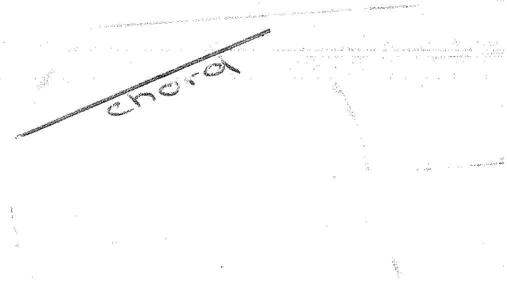
Square

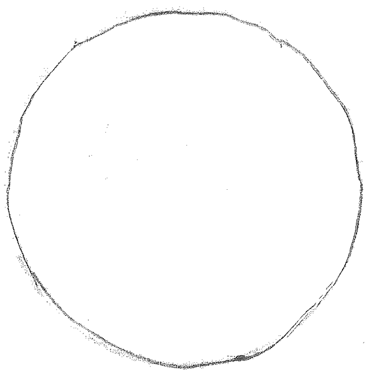


- a parallelogram that is also both a rectangle and a rhombus

Circle Theorems

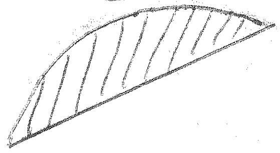


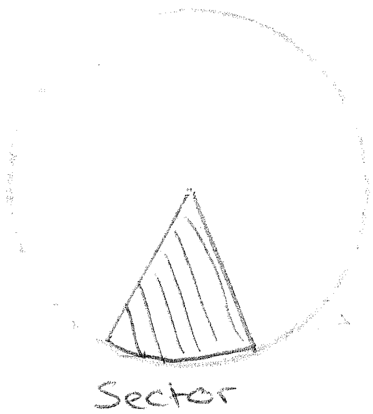


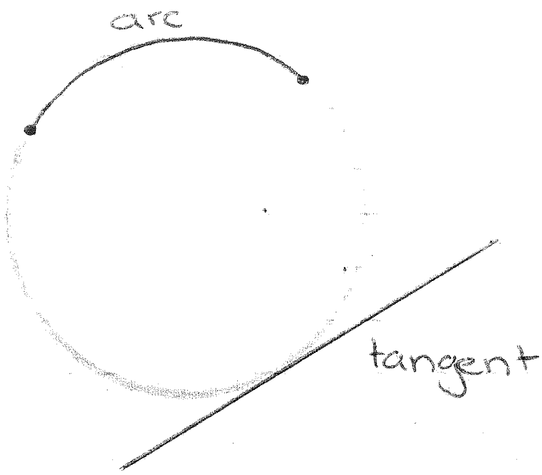


circumference

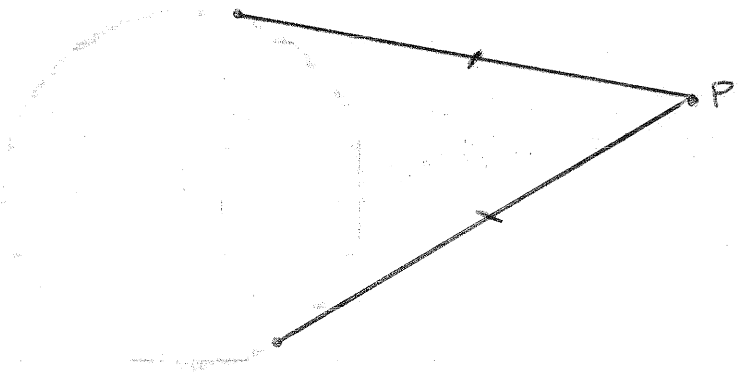
segment



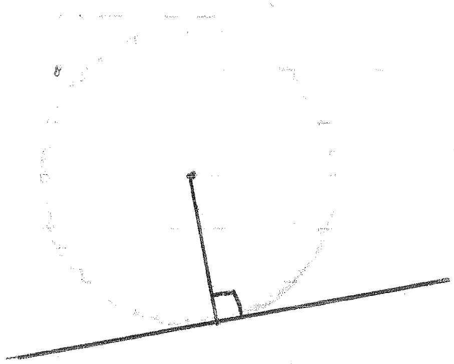




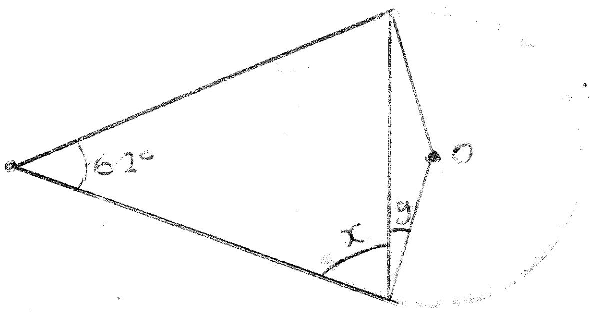
1. Tangents from an external point are equal in length



2. The angle between a radius and a tangent is 90°

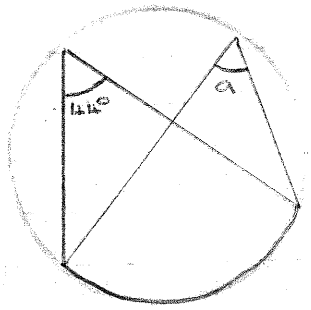


Example



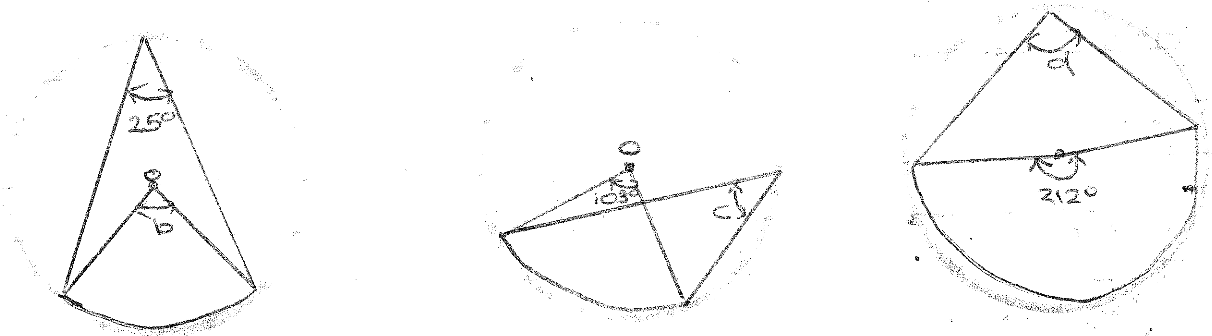
$x = 59°$ (∠sum isos. Δ)
$y = 31°$ (radius + tangent)

3.) Angles formed on the same arc are equal



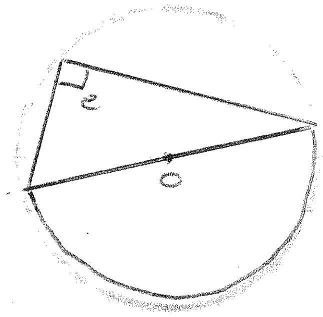
$a = 44°$ (same arc)

4.) The angle at the centre of a circle is twice as big as the angle at the circumference of the circle. (assuming they are on same arc)



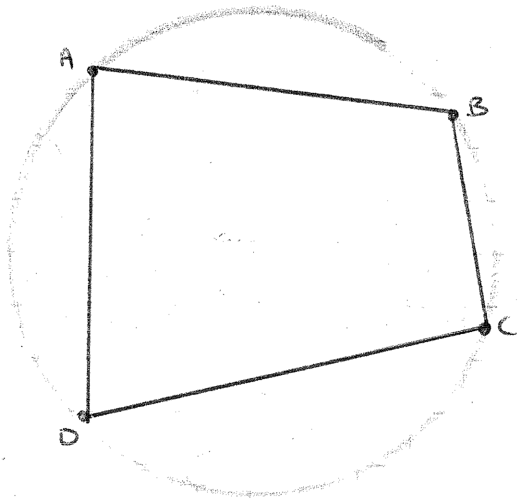
$b = 50°$ (∠ at centre) $c = 54°$ (∠at centre) $d = 106°$ (∠at

5) The angle in a semi circle is 90°.



$e = 90°$ ($\angle$ in semi-circle)

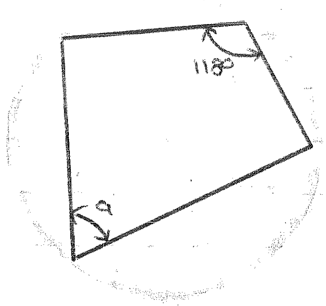
Cyclic Quadrilaterals



A cyclic quadrilateral has all 4 vertices on the circumference of a circle.

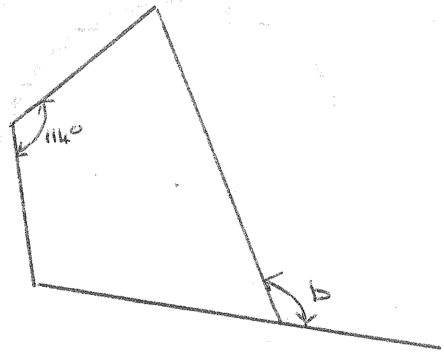
Cyclic quadrilaterals have 2 important properties.

1) The opposite angle of a cyclic quadrilateral are supplementary



$a = 62°$ (opp. $\angle$s cyclic quadrilateral)

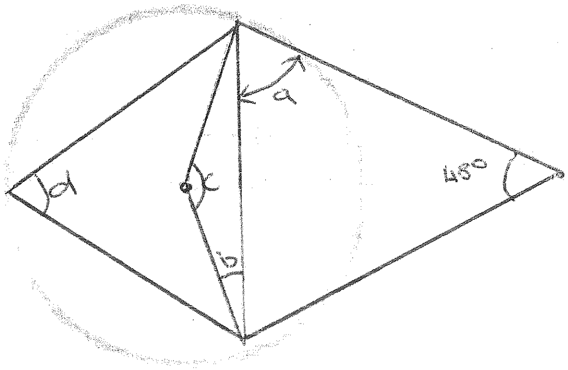
2) The exterior angle of a cyclic quadrilateral equals the interior opposite angle.



b = 114° (ext. ∠ cyclic quad)

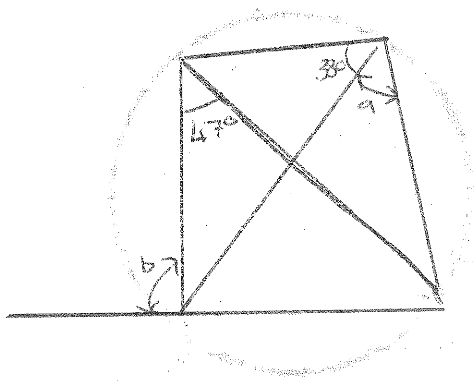
Examples

1)



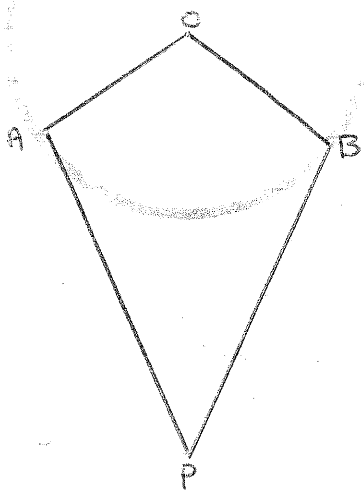
a = 66° (∠ sum isos. △)
b = 24° (radius + tangent)
c = 132° (sum ∠ isos. △)
d = 66° (∠ at centre)

2)



a = 47° (same arc)
b = 85° (ext. ∠ cyclic quad)

3) Explain why PAOB is a cyclic quadrilateral

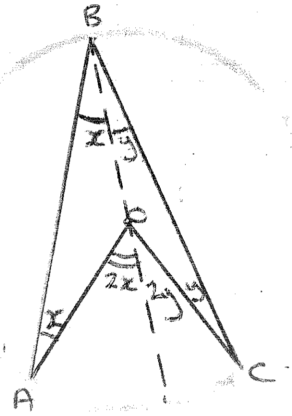


$\angle OAP = 90^\circ$ (radius and tangent)
$\angle OBP = 90^\circ$ (radius and tangent)

$\therefore$ PAOB is a cyclic quadrilateral because opp. $\angle$s add to 180°

Geometrical Proofs

1)



Prove: $\angle AOC = 2 \times \angle ABC$

Proof: let $\angle ABO$ be x

$\angle BAO = x$ (base $\angle$s isos. Δ)

$\angle AOD = 2x$ (ext. $\angle$ of Δ).

Similarly let $\angle OBC$ be y.

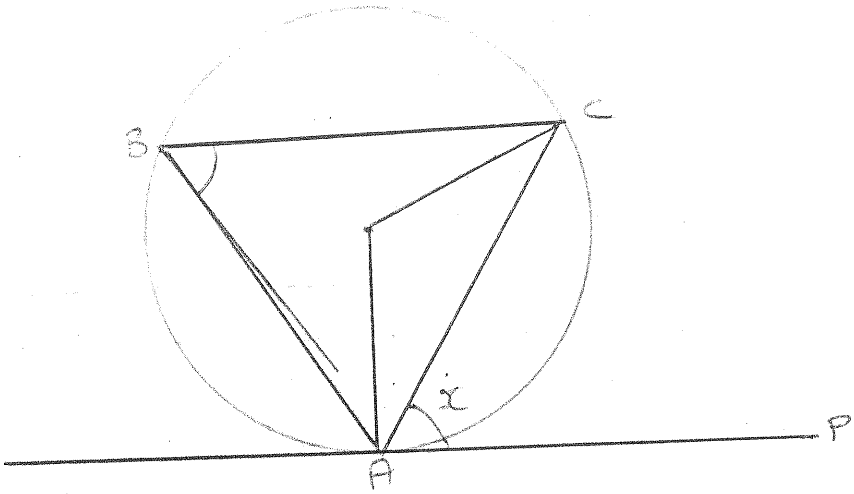
$\therefore \angle COD = 2y$

$\angle AOC = 2x + 2y$

$= 2(x+y)$

$= 2(\angle ABC)$ Q.E.D

2)



Prove: $\angle CAP = \angle ABC$

Proof: let $\angle CAP$ be x

$\angle OAC = 90 - x$ (radius and tangent)
$\angle OCA = 90 - x$ (base $\angle$'s isos Δ)

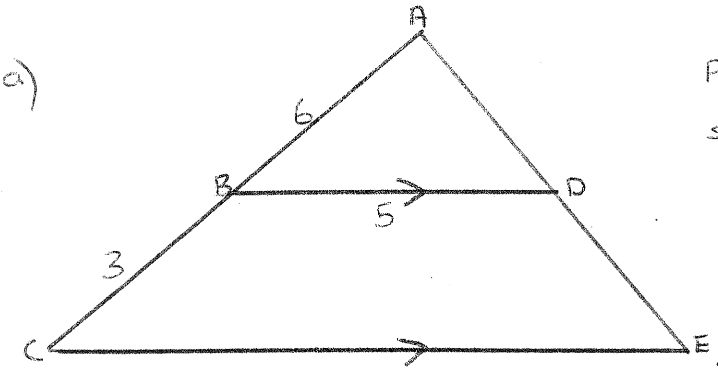
$\angle AOC = 2x$ ($\angle$ sum of Δ)
$\angle ABC = x$ ($\angle$ at centre)

$\therefore \angle CAP = \angle ABC$

Similar Triangles

Similar triangles have the same shape
(ie, the angles are the same)

1) a)



Prove that ΔABD and ΔACE are similar

$\angle ABD = \angle ACE$ (corresp. $\angle$'s, // lines)
$\angle A\ B = \angle AEC$ (corresp. $\angle$'s, // lines)
$\angle BAD = \angle CAE$ (mmon)

$\therefore \Delta$'s ABD and ACE are similar.

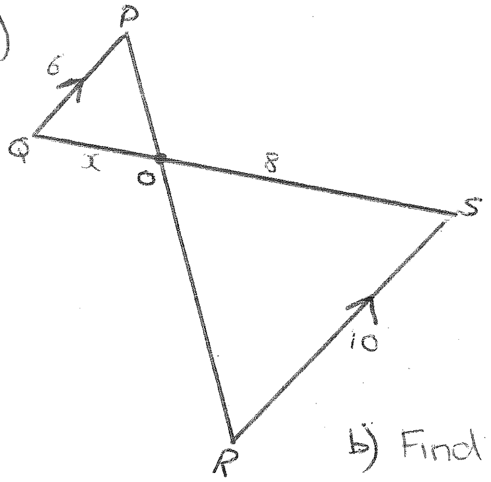
b) Find the length x

$$\frac{x}{5} = \frac{9}{6}$$

$$x = \frac{9}{6} \times 5$$

$$x = 7{\cdot}5$$

2) a)



Prove that the triangles ΔPQO and RCS are similar

$\angle PQO = \angle RSO$ (alt. $\angle$'s, // lines)
$\angle QPO = \angle SRO$ (alt. $\angle$'s, // lines)
$\angle POQ = \angle ROS$ (vert. opp. $\angle$'s)

$\therefore \Delta$s are similar

b) Find length x

$$\frac{x}{8} = \frac{6}{10} \qquad x = \frac{6}{8} \times 8$$

$$x = 4{\cdot}8$$