Achievement Standard 1.6
Probability

Probability is the study of uncertainty

Probabilities are expressed as fractions or decimals between 0 and 1

An event is an outcome of a statistical experiment

e.g. Consider "rolling a dice"
Event A is "getting an even number"
Event B is "getting a six"

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{event A occuring}) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{event B occuring}) = \frac{1}{6}$$

Multivariate Data

Students between years 9-11 at OBHS were surveyed as to which radio station they listen to most often.

	Year 9	Year 10	Year 11	Total
The rock	20	48	65	133
Radio Sport	45	52	60	157
Newstalk ZB	95	40	25	160
Total	160	140	150	450

a) What is the probability that a randomly chosen student was in year 9?

$$P(\text{year } 9) = \frac{160}{450} = \frac{16}{45}$$

b) What is the probability that a randomly chosen student prefers either "Radio Sport" or "Newstalk ZB"?

$$P(\text{Radio Sport or Newstalk ZB}) = \frac{317}{450}$$

c) What is the probability that a randomly chosen student prefers "the rock" given that a year 11 student was chosen?

$$P(\text{The rock, given Yr } 11) = \frac{65}{150} = \frac{13}{30}$$

d) What is the probability that a randomly chosen year 9 student prefers "Radio Sport"?

$$P(\text{Yr 9 student prefers "Radio Sport"}) = \frac{45}{160} = \frac{9}{32}$$

Combined Events

Suppose 32% of students have a drivers license, and 60% of students are girls.

A student is selected randomly
Find P(girl and she has a license)

$= P(\text{girl}) \times P(\text{license})$
$= 0.60 \times 0.32$
$= 0.192$

Further examples:

1) When you buy a packet of weet-bix you sometimes get a free movie pass.

Suppose 1 packet in 3 has a free movie pass.

Blair buys 2 packets
Find the probability he gets 2 free movie passes.

$P(\text{2movie passes}) = P(\text{m and m})$
$= \frac{1}{3} \times \frac{1}{3}$
$= \frac{1}{9}$

2) Sam went camping.
He took 10 tins of food, all of which have lost their labels 6 contained peaches and 4 contained baked beans.
He opens 3 tins for breakfast
Find the probability they all contain peaches.
P(Peaches and peaches and peaches)
$= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$
$= \frac{1}{6}$

3) The probabilities that 3 people, A and B and C, are alive in 10 years time are 0·6, 0·7 and 0·8 respectively.

a) Find the probability that, in ten years time, they are all alive.

$$P(\text{all alive}) = P(A \text{ and } B \text{ and } C)$$
$$= 0{\cdot}6 \times 0{\cdot}7 \times 0{\cdot}8$$

b) Find the probability that, in 10 years time, none are alive

$$P(A' \text{ and } B' \text{ and } C')$$
$$= 0{\cdot}4 \times 0{\cdot}3 \times 0{\cdot}2$$
$$= 0{\cdot}024$$

c) Find the probability that in 10 years time, just one of them is alive.

$$P(\text{one alive}) = P(AB'C' \text{ or } A'BC' \text{ or } A'B'C)$$
$$= 0{\cdot}6 \times 0{\cdot}3 \times 0{\cdot}2 + 0{\cdot}4 \times 0{\cdot}7 \times 0{\cdot}2 + 0{\cdot}4 \times 0{\cdot}3 \times 0{\cdot}8$$
$$= 0{\cdot}188$$

* Some strategies we use for dealing with probability are:

- write out the sample space
- draw a tree diagram

Sample Space

A sample space is a list of the outcomes of a statistical experiment

eg (1) Toss 2 coins

HH, HT, TH, TT

$$P(2 \text{ heads}) = \frac{1}{4}$$

2) A family have 3 children
Write out the sample space

BBB BBG BGB BGG

GBB GBG GGB GGG

P(all girls) $= \frac{1}{8}$
P(exactly 2 boys) $= \frac{3}{8}$

3) Roll 2 dice and add the scores

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a) P(total score of 2) $= \frac{1}{36}$

b) P(total score of 'at least 9) $= \frac{10}{36} = \frac{5}{18}$

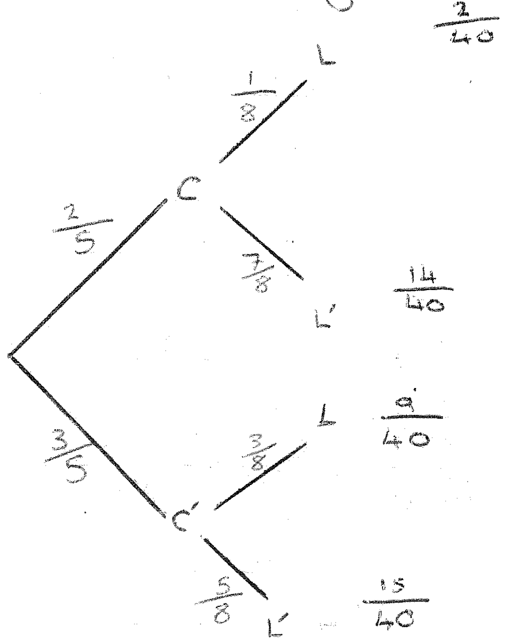
c) P(total score of 7) $= \frac{6}{36} = \frac{1}{6}$

d) I roll 2 dice 1500 times
In how many of these rolls would I expect a total of 7.
$1500 \times \frac{1}{6} = 250$ times

Tree Diagrams

i) Kaspian comes to school by car on $\frac{2}{5}$ of days
When he comes by car he is late on $\frac{1}{8}$ of the days
When he does not come by car he is late on $\frac{3}{8}$ of days.

a) Draw a tree diagram for this situation



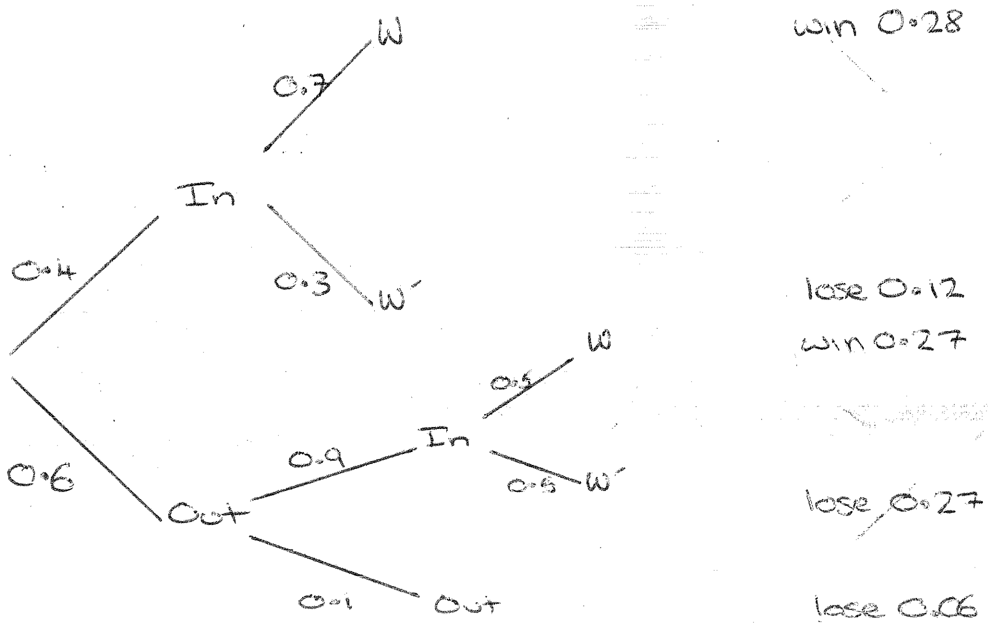
b) Find P(Kaspian comes by car and is late) $= \frac{2}{40} = \frac{1}{20}$

c) Find P(Kaspian is not late to school) $= \frac{14}{40} + \frac{15}{40} = \frac{29}{40}$

d) Given that Kaspian was late to school, find the probability that he came by car.

$$\text{Probability} = \frac{2/40}{11/40} = \frac{2}{11}$$

2) When Adams first serve goes in he has a 70% chance of winning the point. His first serve goes in on 40% of occassions
His second serve goes. in 90% of the time but he only has a 50% chance of winning the point.



a) P(win the point) $= 0.28 + 0.27 = 0.55$

b) P(winning the point if he had 2 serves) $= \frac{27}{60} = \frac{9}{20}$

c) When Adam double-faults, he swears on 25% of time
Find the probability that he will swear.

$$0.6 \times 0.1 \times 0.25 = 0.015$$

Simulations

We can design an experiment to simulate a probability situation. The experiment will involve carrying out a trial a large number of times. We can use the results of the trials to estimate a probability or an average.

1) Weet-bix packets have cards in them. There are 6 different cards in the set. Design a simulation expt. to determine how many packets of Weet-bix you would expect to have to buy to complete the set.

1. Asign each of the cards a number from 1 to 6.
2. Set your calculator to generate random integers from 1 to 6.
3. Generate random integers until we have 1 of every number, and record how many numbers were generated.
4. Repeat this trial a large number of times.
5. Average the results to estimate the number required to complete the set.

2) Mark gets to school on time 3 days out of every 5. The office checked up on late comers 28 days out of 40 last term.
Design a simulation experiment that could be used to predict the probability that Mark is late to school AND the office checked on that day.

1. Set your calculator to generate random integers between 1 and 10
2. Generate random integers in pairs
3. For first number, if 1 to 6 is generated, record L′ (not late) If 7-10 is generated, record L (Late)
4. For second number, if 1 to 7 is generated record C (office checked). If 8-10 is generated, record C′ (did not check)

5. Repeat this simulation a large number of times
6. Count the number of times in trials in which both an L and a C occurred, and call this n.
7. Probability = $\dfrac{n}{\text{total number of trials}}$

3) In a quiz you are asked to answer 2 questions in each round. The first question is on your "specialist subject" and is worth 5 points, while the second question is "general knowledge" and is worth 3 points.
John knows that he gets 60% of his specialist topi questions correct, and 70% of the general knowle questions correct.

The are 10 rounds of questions in the quiz. Design a simulation expt. to estimate the score that John can expect from the 10 rounds of the quiz.

1. Set your calculator to generate random integers between 1 and 10.
2. Generate random integers in pairs to represent the 2 questions in each round.
3. For first number, let 1-6 represent a correct answer, and 7-10 an incorrect answer.
4. If first number generated is 1-6, record 5 points, otherwise record 0 points.
   If second number generated is 1-7, record 3 point otherwise record 0 points.
5. Add up your points for that round
6. Repeat this simulation 10 times since the quiz has 1c rounds

7. Add up the total score from each of the rounds to get an estimate of Johns score.

Achievement Standard 1·2

Graphing

This year we deal with linear graphs (straight lines) and parabolas only.

Linear Graphs

The equation of a straight line is usually written in the form

$$y = mx + c$$

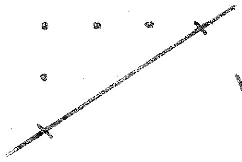
Gradient or slope (m)   y intercept (c)

Slope (or gradient)

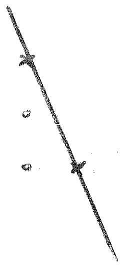
The slope $m = \dfrac{\text{rise}}{\text{run}}$

eg Draw lines with the following slopes

1) $m = \frac{2}{3}$



2) $m = {}^{-}2 = \frac{-2}{1}$



3) $m = -\frac{3}{2} = \frac{-3}{2}$