Quadratic Patterns

(1)

1	1
2	4
3	9
4	16
5	25
⋮	⋮
n	n^2

(2)

1	0
2	1
3	4
4	9
5	16
⋮	
n	$(n-1)^2$

(3)

1	2
2	6
3	12
4	20
5	30
6	42
n	$n(n+1)$

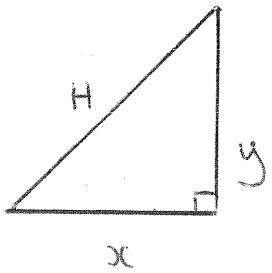
(4)

1	0
2	1
3	3
4	6
5	10
6	15
n	

Achievement standard 1.8

Right-Angled Triangles

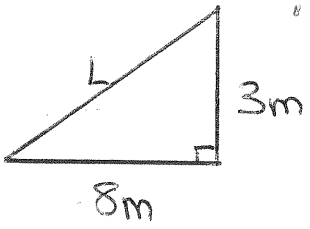
Pythagoras Theorem



$$H^2 = x^2 + y^2$$

Example:

1)



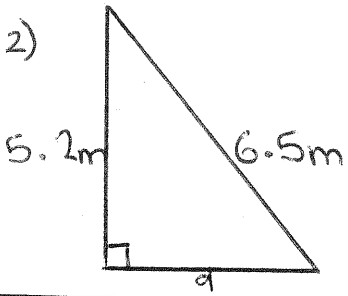
$L^2 = 8^2 + 3^2$

$L^2 = 73$

$L = \sqrt{73}$

$L = 8.5\text{m}$ (1dp)

2)



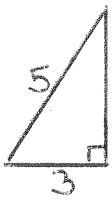
$6.5^2 = d^2 + 5.2^2$

$d^2 = 6.5^2 - 5.2^2$

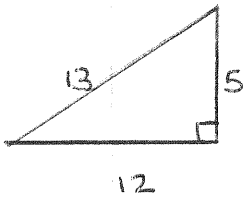
$d^2 = 15.21$

$d = 3.9\text{m}$

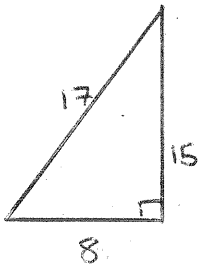
Well known Pythagoras Triples



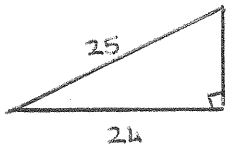
3, 4, 5 △



5, 12, 13 △

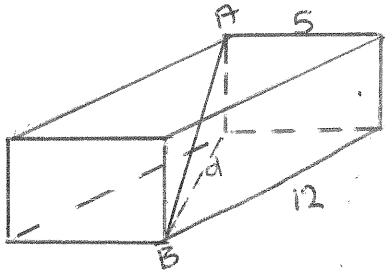


8, 15, 17 △



7, 24, 25 △

3)



Find length AB

$d^2 = 5^2 + 12^2$

$d^2 = 169$

$d = \sqrt{169}$

$d = 13cm$

$AB^2 = 4^2 + 13^2$

$AB^2 = 185$

$AB = \sqrt{185}$

$AB = 13.6cm$ (1dp)

(Note: $AB^2 = 4^2 + 5^2 + 12^2$

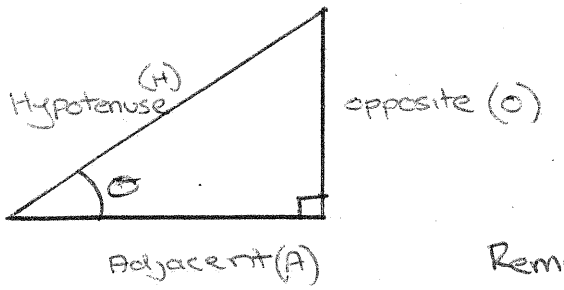
$AB^2 = 185$

$AB = \sqrt{185}$

$AB = 13.6 (1dp)$

∴ Pythagoras Theorem can be applied directly to 3D situations)

Trigonometry



$\sin\theta = \frac{O}{H}$

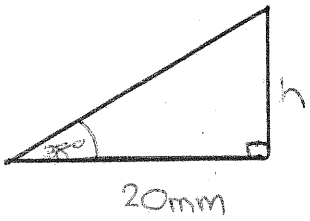
$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

Remember "SOH CAH TOA"

Finding Sides

1)



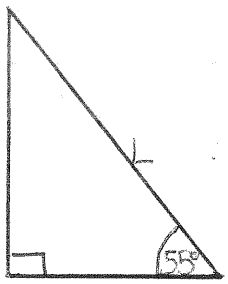
$\tan\theta = \frac{O}{A}$

$\tan 38° = \frac{h}{20}$

$h = 20 \tan 38°$

$h = 15.6m (1dp)$

2)



$\cos\theta = \frac{A}{H}$

$\cos 55° = \frac{4.2}{L}$

$L \cos 55° = 4.2$

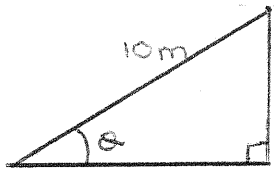
$L = 4.2 / \cos 55°$

$L = 7.32m (2.d.p)$

[redacted]

Finding angles

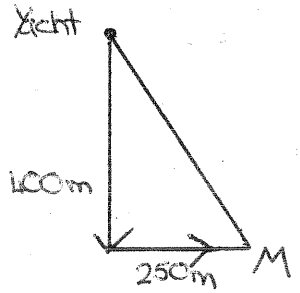
1)



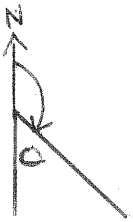
Find angle θ

$$\sin\theta = \frac{O}{H}$$
$$\sin\theta = \frac{3}{10}$$
$$\theta = \sin^{-1}\left(\frac{3}{10}\right)$$
$$\theta = 17.5^\circ \text{ (1dp)}$$

2) Michael fell overboard from a yacht
A strong wind took him 400m south
The tide took him 250 metres east.



Find the bearing of Michael from the yacht



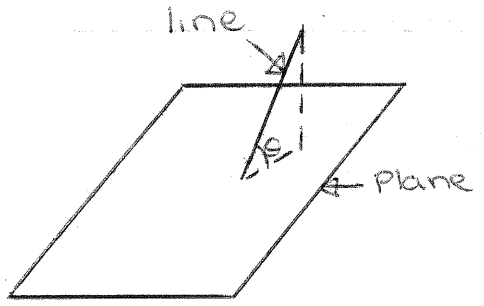
$$\tan\theta = \frac{O}{A}$$
$$\tan\theta = \frac{250}{400}$$
$$\theta = \tan^{-1}\left(\frac{250}{400}\right)$$
$$\theta = 32^\circ$$
$$\therefore$$

Bearing $= 180^\circ - 32^\circ$
$= 148^\circ$

3-D Geometry

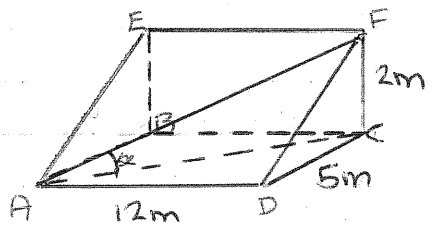
A plane is a flat surface extending indefinitely in all directions.

Angle between line and plane



The angle between a line and a plane is defined as "the angle between the line and its projection on the plane"

Example



A) Find the angle between the line FD and the plane ABCD.

We require $\angle FDC$

$\tan \theta = \frac{2}{5}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 21.8^\circ$ (1dp)

B) Find the angle between the line AF and the plane ABCD.

We require $\angle FAC$

$AC = 13m$ (5, 12, 13)

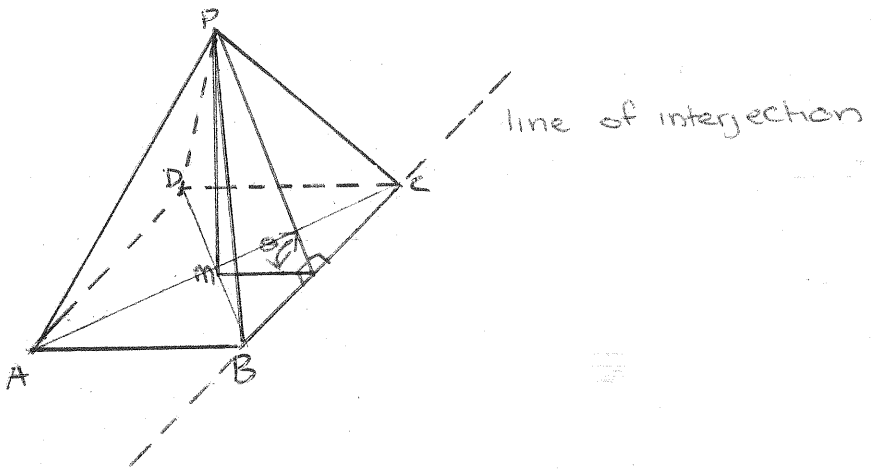
$\tan \theta = \frac{O}{A}$

$\tan \alpha = \frac{2}{13}$

$\alpha = \tan^{-1}\left(\frac{2}{13}\right)$

$\alpha = 8.7^\circ$ (1dp)

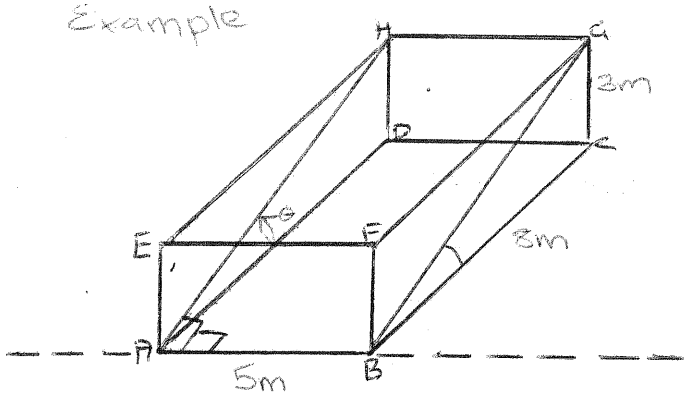
Angle between 2 planes



To find the angle between the plane PBC and the base ABCD

- the planes intersect along line BC
- Draw a line on each of the planes ⊥ to the line
- We require angle θ between these two lines drawn

Example



Calculate the angle between planes ABGH and plane ABCD

$\tan\theta = \frac{O}{A}$

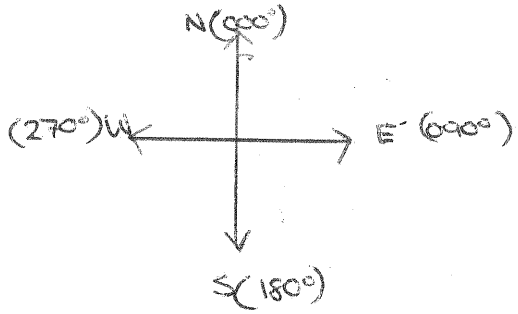
$\tan\theta = \frac{3}{5}$

$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 20.6°$ (1dp)

Bearings

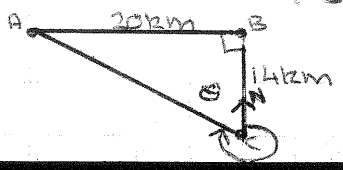
A bearing is a angle (direction) measured clockwise from north.



Example

B is due east of A and 20km away, C is due south of B and 14km away.

Find the bearing of A from C.



$\tan\theta = \frac{O}{A}$ $\quad \theta = 55°$

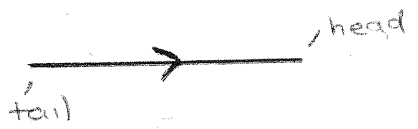
$\tan\theta = \frac{20}{14}$ $\quad$ Bearing $= 360° - 55°$

$\theta = \tan^{-1}\left(\frac{20}{14}\right)$ $\quad = 305°$

Vectors

A vector has a magnitude (size) and a direction.

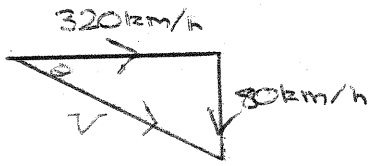
Vectors have arrows on them indicating thier direction.



In a vector diagram vectors are added "head to tail"

Examples

(i) A plane heads due east at 320km/h
A strong wind blows from the north at 80km/h



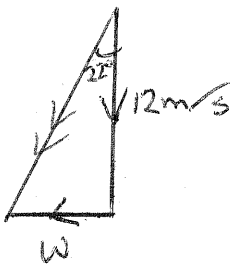
find the resulting speed v and the bearing of the plane.

$$v^2 = 320^2 + 80^2$$
$$v^2 = 108,800$$
$$v = 329.8 \text{ km/h}$$

$$\tan\theta = \frac{80}{320}$$
$$\theta = \tan^{-1}\left(\frac{80}{320}\right)$$
$$\theta = 14^\circ$$

Bearing $= 104^\circ$

(2) Rain falls at 12m/s in still air
They fall at an angle of 22° to the vertical
How strong is the wind?



$\tan\theta = \frac{O}{A}$

$\tan 22° = \frac{W}{12}$

$W = 12 \tan 22°$

$W = 4.8$

The wind is blowing at 4.8m/s