# Quadratics

## Expanding Quadratics

1) $(x+8)(x-3) = x^2 + 5x - 24$

2) $(a-7)(a-5) = a^2 - 12a + 35$

3) $(m+6)(m-6) = m^2 - 36$

4) $(5x-2)(x+4) = 5x^2 + 20x - 2x - 8$
$= 5x^2 + 18x - 8$

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

5) $(x+7)^2 = x^2 + 14x + 49$

6) $(3x-5)^2 = 9x^2 - 30x + 25$

7) $(x+3)(x-2)(x+4) = (x+3)(x^2 + 2x - 8)$
$= x^3 + 2x^2 - 8x + 3x^2 + 6x - 24$
$= x^3 + 5x^2 - 2x - 24$

8) $(x+5)(2x-3)(x+1) =$
$(2x+3)(x^2 + 6x + 5) = 2x^3 + 12x^2 + 10x - 3x^2 - 18x - 15$
$= 2x^3 + 9x^2 - 8x - 15$

# Factorising Quadratics

1. First look for common factors
2. "Difference of 2 squares".
3. Finally look for 2 linear factors

eg 1) $x^2 - 8x = x(x-8)$

2) $3x^2 - 6x = 3x(x-2)$

3) $9x^2 - 4 = (3x+2)(3x-2)$

4) $x^2 + 2x - 35 = (x+7)(x-5)$

5) $x^2 - 2x - 323 = (x+17)(x-19)$

6) $t^2 - 11t + 28 = (t-4)(t-7)$

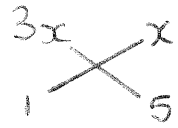
## Factorise Completely

1) $2x^2 - 18 = 2(x^2 - 9)$
$= 2(x+3)(x-3)$

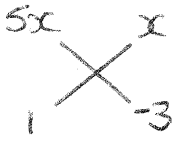
2) $3x^2 + 15x - 42 = 3(x^2 + 5x - 14)$
$= 3(x+7)(x-2)$

# Harder factorisation

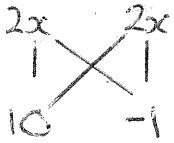
1) $3x^2 + 16x + 5 = (3x + 1)(x + 5)$




2) $5x^2 - 14x - 3 = (5x + 1)(x - 3)$




3) $4x^2 - 13x + 10 = (4x - 5)(x - 2)$


Simplify the following rational expressions by first factorising

1) $\frac{x^2 + 3x - 40}{x-5} = \frac{(x+8)\cancel{(x-5)}}{\cancel{(x-5)}} = x + 8$

2) $\frac{x + 4}{x^2 - 16} = \frac{\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$

3) $\frac{x^2 - 5x + 4}{1 - x^2} = \frac{(x-4)\cancel{(x-1)}^{-1}}{\cancel{(1-x)}(1+x)} = \frac{4-x}{1+x}$

## Completing the square

When completing the square, a quadratic expression is written in the form $(x+a)^2 + b$

Eg

1) $x^2 + 6x + 11 = (x+3)^2 + 2$

2) $x^2 - 10x + 22 = (x-5)^2 - 3$

3) $a^2 + 12a + 9 = (a+6)^2 - 27$

# Quadratic equations

1. First make RHS = 0
2. Factorise
3. Solve for $x$

Eg

1) $(x-8)(x+3)=0$
$x \in \{8, -3\}$

2) $3x(x+6)=0$
$x \in \{0, -6\}$

3) $4(x-1)(x+2)(x-10)$
$x \in \{1, -2, 10\}$

4) $(3x-1)(2x+5)=0$
$x \in \{\frac{1}{3}, -\frac{5}{2}\}$

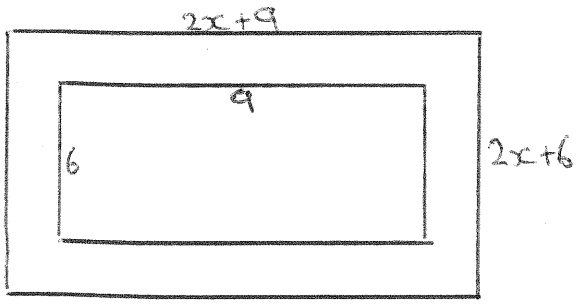
5) $x^2+2x-15=0$
$(x+5)(x-3)=0$
$x \in \{-5, 3\}$

6) $(x+5)^2=9$
$x+5 = \pm 3$
$x = -5 \pm 3$
$x \in \{-2, -8\}$

7) $6=\frac{45}{x}-3x$
$6x = 45-3x^2$
$3x^2+6x-45=0$
$3(x^2+2x-15)=0$
$3(x+5)(x-3)=0$
$x \in \{-5, 3\}$

## Applications

1) Harry has a rectangular lawn which measures 6m by 9m, surrounded by a path of width $x$.
The area of the path is $34m^2$
Find the width $x$ of the path.




$(2x+9)(2x+6)-54=34$
$4x^2+12x+18x+54-54=34$
$4x^2+30x=34$
$4x^2+30x-34=0$
$x \in \{-8.5, 1\}$
Width of path is 1m

2) The height of the sky rocket fired vertically upwards at 50m/s is given by.

$h = 50t - 5t^2$

(where $t$ = time in seconds)

find the two times that the rocket is at a height of 105m

$$105 = 50t - 5t^2$$

$$5t^2 - 50t + 105 = 0$$

$$t \in \{7, 3\}$$

It is at a height of 105m after 3 seconds and after 7 seconds.

# Quadratic formula

When quadratic equations cannot be factorised, or are difficult to factorise, we can solve the equation by either:

- using a quadratic formula
- using a graphics calculator

To solve the equation $ax^2 + bx + c = 0$ we use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

1) Solve $2x^2 + 4x + 1 = 0$

$a = 2 \quad b = 4 \quad c = 1$

$$b^2 - 4ac = 16 - 4 \times 2 \times 1$$
$$= 16 - 8$$
$$= 8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-4 \pm \sqrt{8}}{4}$$

$$x \in \{-0.29, -1.71\}$$

2) Solve $2x^2 - 7x + 2 = 0$

$a = 2 \quad b = -7 \quad c = 2$

$$b^2 - 4ac = 49 - 4 \times 2 \times 2$$
$$= 49 - 16$$
$$= 33$$

$$x = \frac{7 \pm \sqrt{33}}{4}$$

$$x \in \{3.19, 0.31\}$$

3) $5t = 4 - 3t^2$

$3t^2 + 5t - 4 = 0$

$a = 3 \quad b = 5 \quad c = -4$

$b^2 - 4ac = 25 - 4 \times 3 \times -4$

$= 25 - -48$

$= 73$

$x = \frac{-5 \pm \sqrt{73}}{6}$

## Graphics calculator

To solve a quadratic equation using a G.C. the equation must first be written in the form $ax^2 + bx + c =$

Then from [MENU] select [EQUA]

The select "polynomial"

For a quadratic, degree of our polynomial is 2. The co-efficients a, b and c must now be entered in the matrix

Then select [SOLV]

e.g. To solve $6.9x = 3 - 0.72x^2$

Re-write as $0.72x^2 + 6.9x - 3$

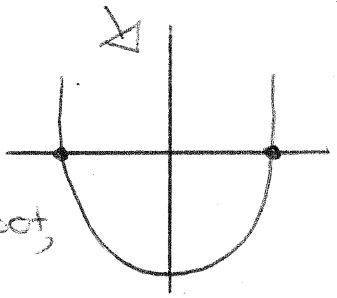
$[0.72 \quad 6.9 \quad -3]$

$x \in \{0.42, -10\}$

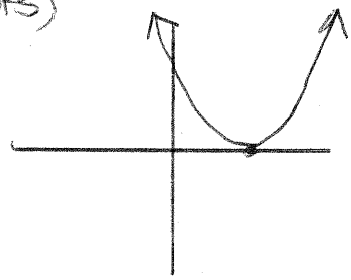
# Discriminant

$b^2 - 4ac$ is called the discriminant

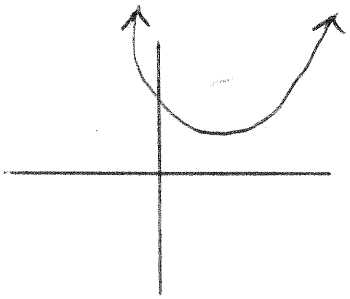
If $b^2 - 4ac > 0$, the equation has 2 real roots



If $b^2 - 4ac = 0$, the equation has 1 real root, (or 2 equal roots)



If $b^2 - 4ac < 0$, the equation has no real roots



1) State the nature of the roots of

$$2x^2 + 3x + 5 = 0$$

$$a = 2 \quad b = 3 \quad c = 5$$

$$b^2 - 4ac = 9 - 4 \times 2 \times 5$$
$$= 9 - 40$$
$$= -31$$

No real roots exist

2) For what values of p does the equation
$px^2 + 2x + 5 = 0$
have 2 real roots?

$a = p \quad b = 2 \quad c = 5$

Require $b^2 - 4ac > 0$

$(2)^2 - 4 \cdot p \cdot 5 > 0$

$4 - 20p > 0$

$-20p > -4$

$p < 0.2$

3) For what values of q does the equation $\frac{4x^2}{5} + 3x = q$ have no real roots?

$4x^2 + 15x - 5q = 0$

$a = 4 \quad b = 15 \quad c = -5q$

Require $b^2 - 4ac < 0$

$(15)^2 - 4 \times 4 \times -5q < 0$

$225 - -80q < 0$

$225 + 80q < 0$

$80q < -225$

$q < -2,8125$