

## Exponents

### Laws of Exponents

$$1. a^m \times a^n = a^{m+n}$$

$$2. \frac{a^m}{a^n} = a^{m-n}$$

$$3. (a^m)^n = a^{mn}$$

$$4. a^{-n} = \frac{1}{a^n}$$

$$5. a^{\frac{1}{n}} = \sqrt[n]{a}$$

Examples

$$1) 4a^3 \times 5a^2 = 20a^5$$

$$2) \frac{20b^5}{15b^2} = \frac{4}{3}b^3 \quad \text{or} \quad \frac{4b^3}{3}$$

$$3) (2x^4)^3 = 8x^{12}$$

$$\text{Evaluate: } 1) 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$2) 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$3) \left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$$

Write using positive exponents

$$1) 5x^{-2} = \frac{5}{x^2}$$

$$2) 4a^{-3} = \frac{4}{a^3}$$

Write in the form  $kx^n$

$$1) \frac{3}{x^4} = 3x^{-4}$$

$$2) \frac{5}{a^2} = 5a^{-2}$$

Write the following using surds:

$$1) a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$2) 5x^{\frac{1}{2}} = 5\sqrt{x}$$

$$3) x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

$$4) x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$

Write using exponents

$$1) \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$2) \sqrt[4]{x^3} = x^{\frac{3}{4}}$$

$$3) \frac{1}{\sqrt{x^5}} = x^{-\frac{5}{2}}$$

Evaluate

$$1) 4^{\frac{3}{2}} = 8$$

$$2) 32^{\frac{3}{5}} = 8$$

$$3) 9^{-\frac{3}{2}} = \frac{1}{27}$$

$$4) \left(\frac{9}{16}\right)^{-\frac{3}{2}} = \frac{64}{27}$$

## Logarithms

$$1000 = 10^3 \begin{matrix} \uparrow & \uparrow \\ \text{a number} & \text{base} \end{matrix} \begin{matrix} \text{power or} \\ \text{logarithm} \end{matrix}$$

Every number can be written as a power of 10.

$$\text{e.g. } 527 = 10^{2.721810615}$$

The log using base 10 of the number 527 is 2.721810615

$$\text{i.e. } \log_{10} 527 = 2.721810615$$

$$\text{If } y = b^x, \text{ then } \log_b y = x$$

Examples: Write the following exponential statements as log statements.

$$1) 8 = 2^3 \rightarrow \log_2 8 = 3$$

$$2) \frac{1}{4} = 8^{-2/3} \rightarrow \log_8 \left(\frac{1}{4}\right) = -\frac{2}{3}$$

Write the following log statements as exponential statements.

$$1) \log_5 125 = 3 \rightarrow 125 = 5^3$$

$$2) \log_2 256 = 8 \rightarrow 2^8 = 256$$

## Log equations

$$\begin{aligned} 1) \log_x 243 &= 5 \\ x^5 &= 243 \\ x &= \sqrt[5]{243} \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 2) \log_2 x &= 5 \\ 2^5 &= x \\ x &= 32 \end{aligned}$$

$$\begin{aligned} 3) \log_2 128 &= x \\ 2^x &= 128 \\ x &= 7 \end{aligned}$$

$$4) \log_x 243 = \frac{5}{3} \quad x^{\frac{5}{3}} = 243 \quad x = \sqrt[3]{243} \quad x = 27$$

## Laws of logarithms

$$1) \underline{\log a + \log b = \log ab}$$

$$2) \underline{\log a - \log b = \log \left(\frac{a}{b}\right)}$$

$$3) \underline{\log a^n = n \log a}$$

Write as a log of a single number

$$1) \log 8 + \log 3 = \log 24$$

$$2) \log 20 - \log 5 = \log 4$$

$$3) 3 \log 2 = \log 2^3 = \log 8$$

$$4) \log 6 - \log 8 + \log 12 = \log \left(\frac{6 \times 12}{8}\right) \\ = \log 9$$

$$5) 3 \log 5 - \frac{1}{2} \log 100 + 3 \log 2 = \log 5^3 - \log 100 \\ = \log 125 - \log 100 + \\ = \log \left(\frac{125 \times 8}{10}\right) = \log 1$$

## Exponent Equations

1) Solve  $3^x = 20$

Take logs of both sides.

$$\log 3^x = \log 20$$

$$x \cdot \log 3 = \log 20$$

$$x = \frac{\log 20}{\log 3}$$

$$x = 2.73$$

- 2) Tat invests \$5000 at 6% paid once annually.  
The amount of money Tat has after  $n$  years is given by  $A = 5000 \times 1.06^n$ .

After how many years will he have \$20000

$$20000 = 5000 \times 1.06^n$$

$$4 = 1.06^n$$

$$\log 4 = \log 1.06^n$$

$$\log 4 = n \log 1.06$$

$$n = \log 4 / \log 1.06 = 23.8 \text{ years}$$

Tat must wait 24 years.

- 3) After a party Hugo's blood alcohol level is 320.  
It decays exponentially according to

$$C = 320 \times 0.8^t$$

Where  $C$  = blood alcohol level

$t$  = time in hours

Calculate how long Hugo must wait until his level drops to 80.

$$80 = 320 \times 0.8^t$$

$$.25 = 0.8^t$$

$$\log(.25) = \log(0.8^t)$$

$$t = \frac{\log .25}{\log .8} = 6.2 \text{ hours}$$