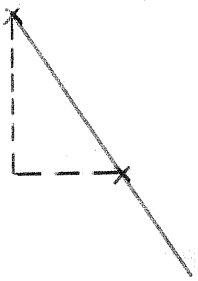
Achievement Standard 2.4
Co-ordinate Geometry

Gradient / Slope

$$\text{Slope } m = \frac{\Delta y}{\Delta x}$$

1) slope $m = \frac{1}{2}$

2) Slope $m = \frac{-3}{2}$



Slope between 2 points

Find the slope between A(-1, 5) and B(5, 2)

$\Delta x = 6$

$\Delta y = -3$

$$\text{slope } m = \frac{\Delta y}{\Delta x} = -\frac{3}{6} = -\frac{1}{2}$$

2) Find p if the gradient of the line joining A(3,p) to B(-1,10) is $m = -2$.

$\Delta x = -4$

$\Delta y =$

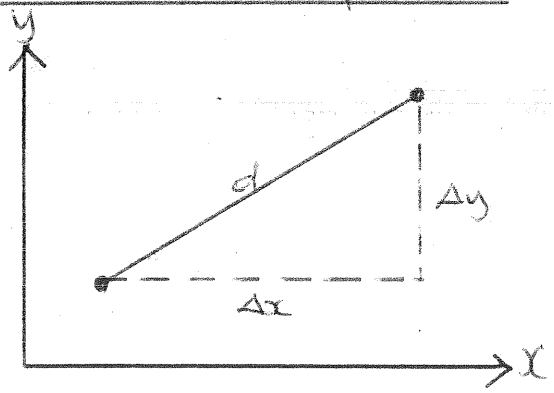
$m = \frac{\Delta y}{\Delta x}$

$-2 = \frac{\Delta y}{-4}$

$\Delta y = 8$

$\therefore p = 2$

Distance between 2 points



The distance between 2 points is given by the formula

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

e.g. 1) Find the distance between (-1, 5) and (5, 2)

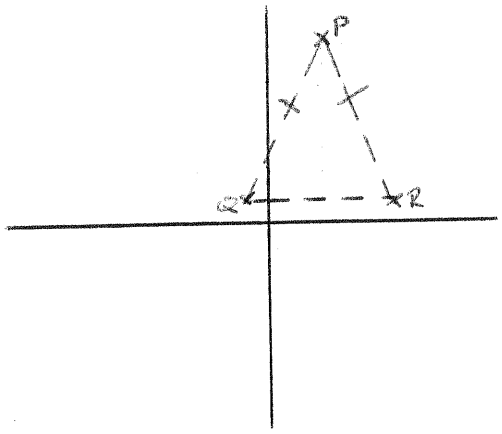
$\Delta x = 6$

$\Delta y = -3$

$d = \sqrt{6^2 + (-3)^2}$

$d = \sqrt{45}$ or $3\sqrt{5}$ or 6.7

7) Show that ΔPQR with vertices $P(2,7)$, $Q(-1,1)$ and $R(5,1)$ is isosceles. Name the pair of equal sides.



PQ: $\Delta x = -3$
$\Delta y = -6$

$d = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$

PR: $\Delta x = 3$
$\Delta y = -6$

$d = \sqrt{3^2 + (-6)^2} = \sqrt{45}$

$\therefore \Delta PQR$ is isosceles

Collinear Points

Points A, B and C are collinear if they lie on a straight l

Eg 1) Show that $A(-3,8)$ and $B(1,3)$ and $C(13,-12)$ are collinear.

For AB: $\Delta x = 4$
$\Delta y = -5$
Slope $m = \frac{-5}{4}$

For BC: $\Delta x = 12$
$\Delta y = -15$
slope $m = \frac{-15}{12} = -\frac{5}{4}$

$\therefore m_{AB} = m_{BC}$, so points are collinear.

2) Points $P(-5,3)$ and $Q(1,1)$ and $R(k,-3)$ are collinear. Find value of k.

$PQ: \Delta x = 6$

$\Delta y = -2$

$\text{Slope } m = PQ = \frac{-2}{6} = -\frac{1}{3}$

$QR: \Delta x =$

$\Delta y = -4$

$m_{QR} = \frac{-4}{\Delta x}$

$-\frac{1}{3} = \frac{-4}{\Delta x}$

$-\frac{\Delta x}{3} = -4$

$-\Delta x = -12$

$\Delta x = 12$

$\therefore k = 13$

Midpoint

If $A(x,y)$ and $B(x_2,y_2)$ then the midpoint of AB is given by $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

1) eg Find the midpoint of the line joining $P(-7,-4)$ and $Q(1,12)$

Midpoint is $M\left(\frac{-7+1}{2}, \frac{-4+12}{2}\right)$

$= M(4,4)$

2) $\overline{AB}$ is the diameter of a circle. If $A(3,2)$ and $B(9,-6)$, find the equation of the circle.

For AB: $\Delta x = 6$

$\Delta y = -8$

$d = \sqrt{6^2+(-8)^2}$

$d = 10$

$\therefore$ radius of circle is 5

Midpoint of AB is $\left(\frac{3+9}{2}, \frac{2+-6}{2}\right) = (6,-2)$

Equation of circle is $(x-6)^2+(y+2)^2 = 25$

Gradient intercept method

Recall $y = mx + c$

- m: gradient or slope
- c: intercept

i) Write the equation of a line whose slope is -2 and which cuts y axis at 5

$y = -2x + 5$

General Equation of a straight line

All straight lines can be written in the general form $ax + by + c = 0$ where a, b and c are integers (the x term should be positive)

eg Write $y = \frac{5}{8}x + 3$ in the general form of straight line

$8y = 5x + 24$

Equation is $5x - 8y + 24 = 0$

Point - Slope Equation

If we know the slope m of a line, and we know that the line passes through point (x, y), then the equation of the line is.

$$y - y_1 = m(x - x_1)$$

1) Find the equation of the line whose slope is 2, and which passes through the point $(3, 8)$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= 2(x - 3) \\ y - 8 &= 2x - 6 \\ y &= 2x + 2 \end{aligned}$$

2) Find the equation of the line whose slope is $\frac{2}{3}$ and which passes through $(5, -2)$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - -2 &= \tfrac{2}{3}(x - 5) \\ y + 2 &= \tfrac{2}{3}(x - 5) \\ 3y + 6 &= 2(x - 5) \\ 3y + 6 &= 2x - 10 \\ 2x - 3y - 16 &= 0 \end{aligned}$$

3) Find the equation of the line joining $(5, 1)$ and $(9, 13)$

$$\begin{aligned} \Delta x &= 4 \\ \Delta y &= 12 \\ m &= \tfrac{12}{4} = 3 \end{aligned}$$

Using $y - y_1 = m(x - x_1)$

$$\begin{aligned} y - 1 &= 3(x - 5) \\ y - 1 &= 3x - 15 \\ y &= 3x - 14 \end{aligned}$$

Parallel lines

Parallel lines have the same slope

1) Find the equation of the line parallel to $y = \frac{1}{2}x + 3$ and passing through point $(-5, 4)$

$m = \frac{1}{2}$

$m_{//} = \frac{1}{2}$

$y - 4 = \frac{1}{2}(x - -5)$

$2y - 8 = x + 5$

$0 = x - 2y + 13$

2) Find the equation of the line parallel to $3x - 2y + 5 = 0$ and passing through $(10, 1)$

For $3x - 2y + 5 = 0$

$3x + 5 = 2y$

$\frac{3}{2}x + \frac{5}{2} = y$

$m = \frac{3}{2}$

$m_{//} = \frac{3}{2}$

$\therefore y - 1 = \frac{3}{2}(x - 10)$

$2y - 2 = 3(x - 10)$

$2y - 2 = 3x - 30$

$3x - 2y - 28 = 0$

Perpindicular lines

If a line has slope $m = \frac{a}{b}$, then a perpindicular line has slope

$$m_{\perp} = -\frac{b}{a}$$

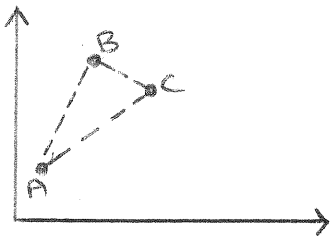
Eg 1) Find the equation of a line perpindicular to $y = \frac{2}{3}x + 1$ and passing through $(8, 3)$

$$m = \frac{2}{3}$$
$$m_{\perp} = -\frac{3}{2}$$

Using $y - y_1 = m(x - x_1)$

$$y - 3 = -\frac{3}{2}(x - 8)$$
$$2y - 6 = -3(x - 8)$$
$$2y - 6 = -3x + 24$$
$$3x + 2y - 30 = 0$$

2) If 2 lines are perpindicular to each other, then $m_1 \times m_2 = -1$



ABC is a Δ with vertices A(2, 1) B(6, 3) and C(5, 5) Show that ΔABC is a right angled triangle.

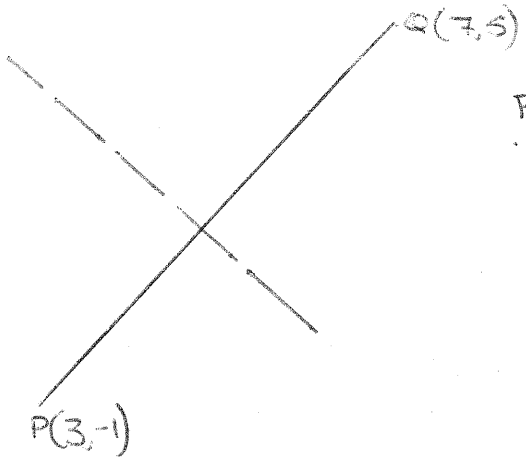
For AB = $\Delta x = 4$
$\Delta y = 2$
$m_{AB} = \frac{2}{4} = \frac{1}{2}$

For BC = $\Delta x = -1$
$\Delta y = 2$
$m_{BC} = \frac{2}{-1} = -2$

$$m_{AB} \times m_{BC} = \frac{1}{2} \times -2$$
$$= -1$$

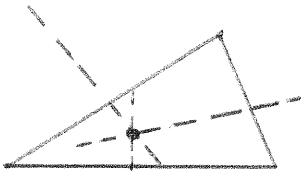
$\therefore$ sides AB and BC are $\perp$

3)



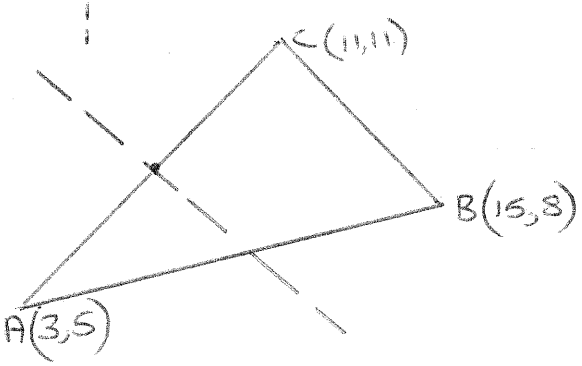
Find the equation

Perpindicular bisector



The 3 perpindicular bisectors of a △ meet at the circumcentre

Eg



Find the equation of the perpindicular bisector of AC

Midpoint of AC = (7,8)

$\Delta x = 8$

$\Delta y = 6$

$m_{AC} = \frac{6}{8} = \frac{3}{4}$

$m_{\perp} = -\frac{4}{3}$

$y - 8 = -\frac{4}{3}(x-7)$

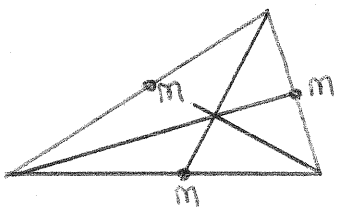
$3y - 24 = -4(x-7)$

$3y - 24 = -4x + 28$

$4x + 3y - 52 = 0$

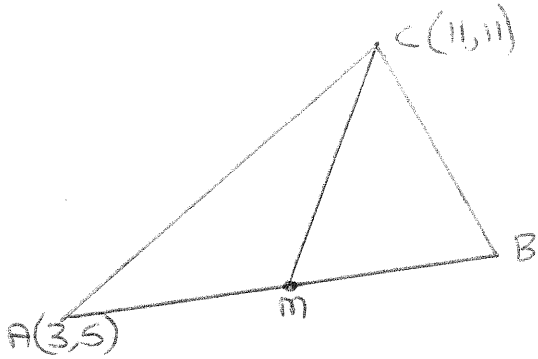
Medians

A <u>median</u> is a line from a vertex to the <u>midpoint</u> of the opposite side.



The 3 medians intersect at the <u>centroid</u>

eg



Find the equation of the median thro point C

Midpoint of AB is (9, 6.5)

Slope of line through M(9, 6.5) and C(11, 11)

$\Delta x = 2$

$\Delta y = 4.5$

$m = \frac{4.5}{2} = \frac{9}{4}$

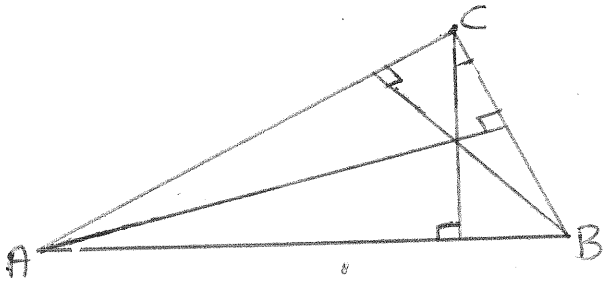
Using $y - y_1 = m(x - x_1)$

$y - 11 = \frac{9}{4}(x - 11)$

$4y - 44 = 9x - 99$

$0 = 9x - 4y - 55$

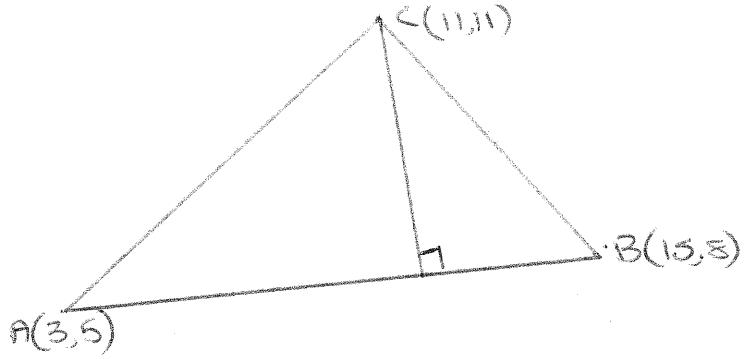
Altitudes



An altitude runs from a vertex to meet the opposite side at 90°

The 3 altitudes must meet at the orthocent

eg



Find the equation of the altitude through point C.

For AB = $\Delta x = 12$

$\Delta y = 3$

$m = \frac{3}{12} = \frac{1}{4}$

$m_{\perp} = -\frac{4}{1}$

Using $y - y_1 = m(x - x_1)$

$y - 11 = -4(x - 11)$

$y - 11 = -4x + 44$

$y = -4x + 55$