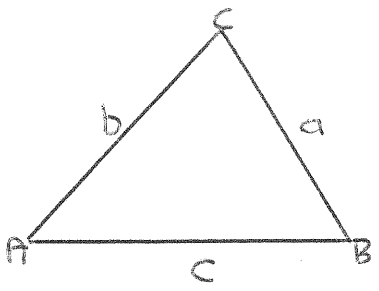
Achievement Standard 2.8

Practical Trigonometry

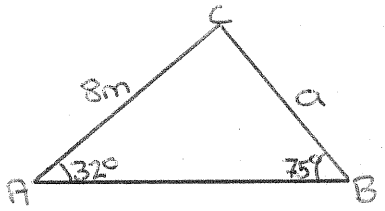
Sine rule



For finding sides:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Examples 1)



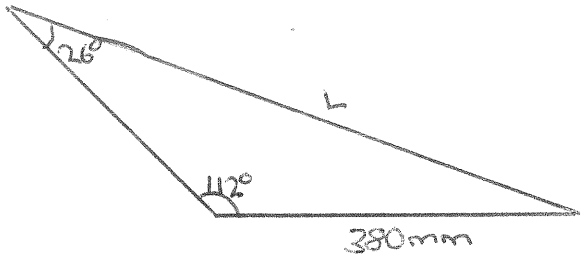
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 32^\circ} \quad \frac{8}{\sin 75^\circ}$$

$$a = \frac{8 \sin 32^\circ}{\sin 75^\circ}$$

$$a = 4{\cdot}4\text{m}$$

2)



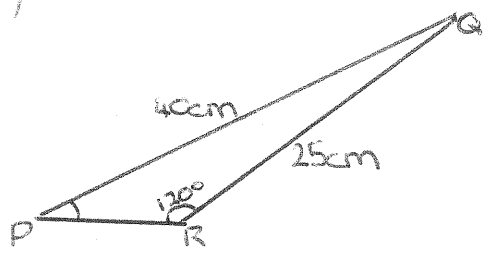
$$\frac{L}{\sin 112} = \frac{380}{\sin 26}$$

$$L = \frac{380 \sin 112^\circ}{\sin 26^\circ}$$

$$L = 804\text{mm}$$

For finding angles: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

1)



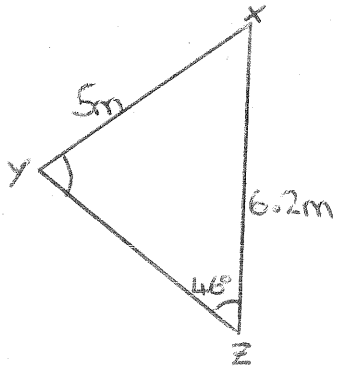
Find $\angle P$

$$\frac{\sin P}{25} = \frac{\sin 120}{40}$$

$$\sin P = \frac{25 \sin 120^\circ}{40}$$

$$P = 32.8^\circ \text{ (1d.p)}$$

2)

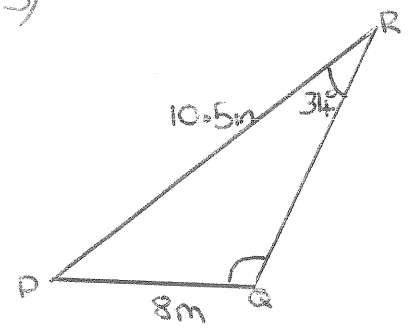


$$\frac{\sin Y}{6.2} = \frac{\sin 46^\circ}{5}$$

$$\sin Y = \frac{6.2 \sin 46}{5}$$

$$Y = 63.1^\circ \text{ (1dp)}$$

3)



Find the obtuse angle $\hat{Q}$

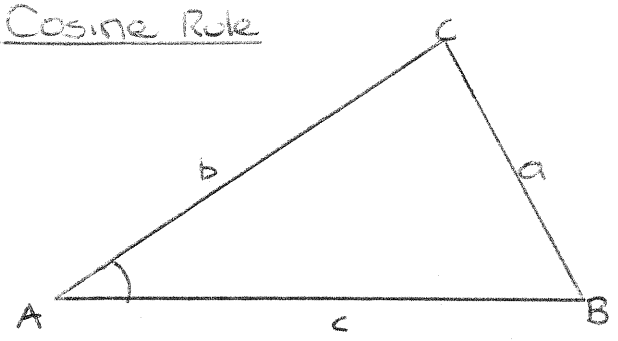
$$\frac{\sin\theta}{10.5} \quad \frac{\sin 34^\circ}{8}$$

$$\sin Q = \frac{\sin 34^\circ}{8} \times 10.5$$

$$Q = 180^\circ - 47.2^\circ$$

$$Q = 132.8^\circ$$

Cosine Rule



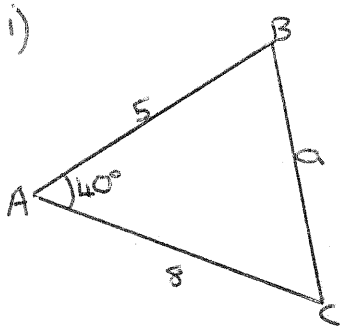
For finding sides:

* We must know the other 2 sides and the included (opposite) angle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Examples:

1)



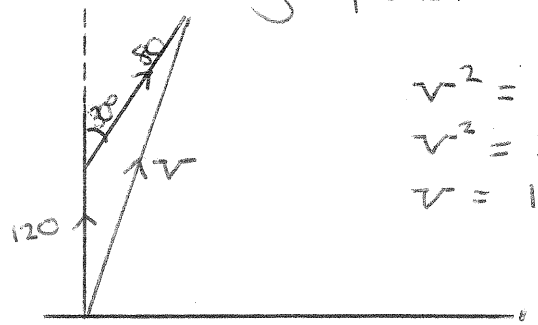
$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 40°$

$a^2 = 27.72$

$a = 5.3$ (1dp)

2) A plane is flying north at 120km/h. A wind is blowing on a bearing of 030° at a speed of 80km/h. Find the resulting speed of the plane



$v^2 = 120^2 + 80^2 - 2 \times 120 \times 80 \times \cos 150°$

$v^2 = 37427.7$

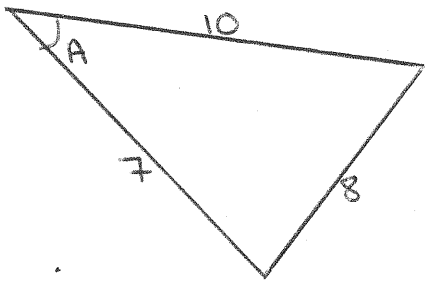
$v = 193.5$ km/h

For finding angles:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Examples

1)



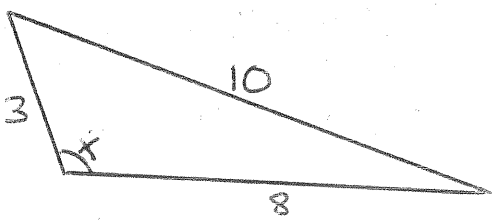
$$\cos A = \frac{10^2 + 7^2 - 8^2}{2 \times 10 \times 7}$$

$$\cos A = \frac{85}{140}$$

$$A = \cos^{-1}\left(\frac{85}{140}\right)$$

$$A = 52.6°$$

2)



$$\cos X = \frac{8^2 + 3^2 - 10^2}{2 \times 8 \times 3}$$

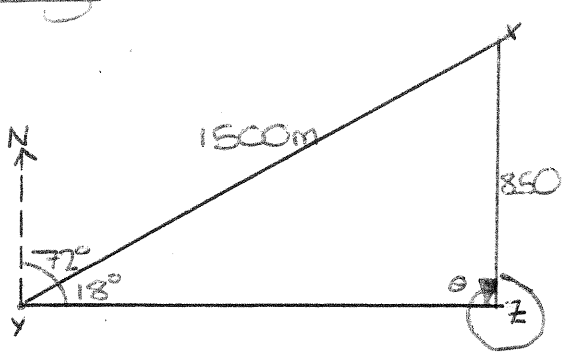
$$\cos X = \frac{-27}{48}$$

$$X = \cos^{-1}\left(\frac{-27}{48}\right)$$

$$X = 124.2°$$

Bearings

1)



Bearing of X from Y is 072°
Distance XX is 1500m
Distance XZ is 850m
Z is due east of Y

Find the bearing of X from Z

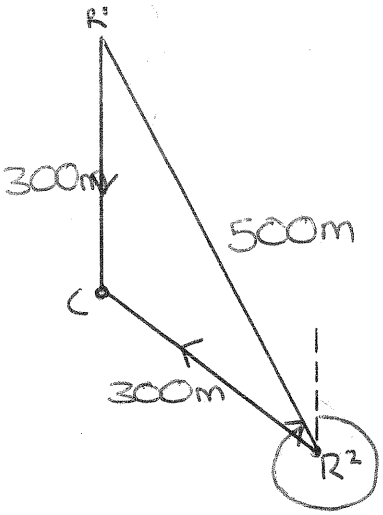
$$\frac{\sin\theta}{1500} = \frac{\sin 18^\circ}{850}$$

$$\sin\theta = \frac{1500 \sin 18^\circ}{850}$$

$$\theta = 33^\circ$$

$$\text{Bearing} = 270 + 33$$
$$= 303^\circ$$

2)



Find the bearing of R^1 from R^2

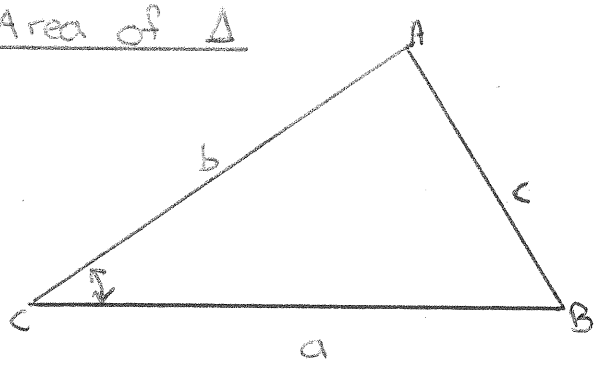
$$\cos A = \frac{300^2 + 500^2 - 300^2}{2 \times 300 \times 500}$$

$$\cos A = \frac{250000}{300000}$$

$$A = 34^\circ$$

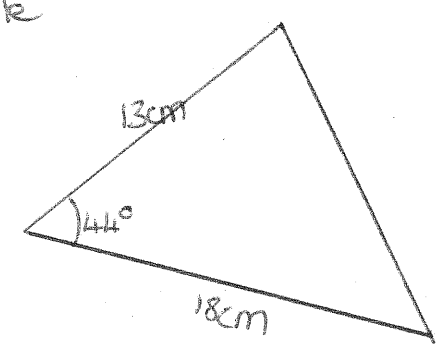
$$\text{Bearing} = 360^\circ - 34^\circ$$
$$= 326^\circ$$

Area of Δ



$$\text{Area } \Delta = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}ac\sin B$$
$$= \frac{1}{2}bc\sin A$$

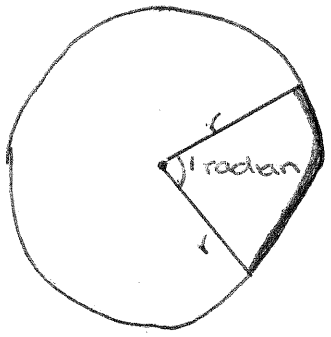
Example



$$\text{Area } \Delta = \frac{1}{2} \times 13 \times 18 \times \sin 44$$
$$= 81.3\text{cm}^2$$

Radians

Angles are usually measured in degrees or radians



1 radian is the angle formed in a sector when the arc length is equal to the radius

$180° = \pi$ radians

It follows from this, that

π radians $= 180°$

1 radian $= \frac{180°}{\pi} = 57.3°$

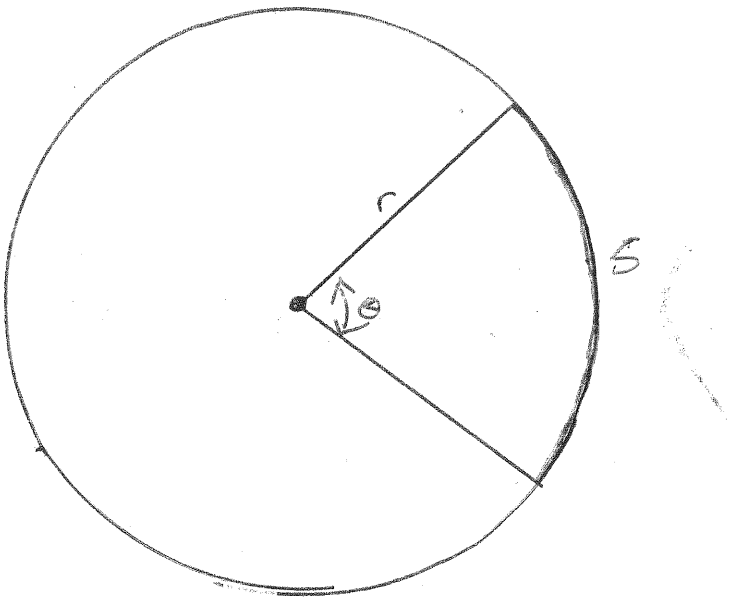
Convert the following angles into radians

1) $90° = \frac{\pi}{2}$
2) $60° = \frac{\pi}{3}$
3) $120° = \frac{2\pi}{3}$
4) $30° = \frac{\pi}{6}$
5) $1° = \frac{\pi}{180}$
6) $15° = \frac{15\pi}{180} = \frac{\pi}{12}$

Convert the following into degrees

1) $\frac{5\pi}{6} = 150°$
2) $3.72 = 3.72\left(\frac{180}{\pi}\right)$
$= 213.1°$

Arc Length

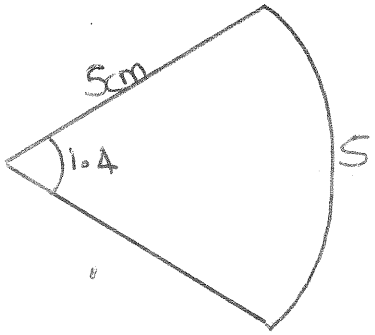


The arc length S is given by

$$S = r\theta$$

N.B. θ must be in radians

1)



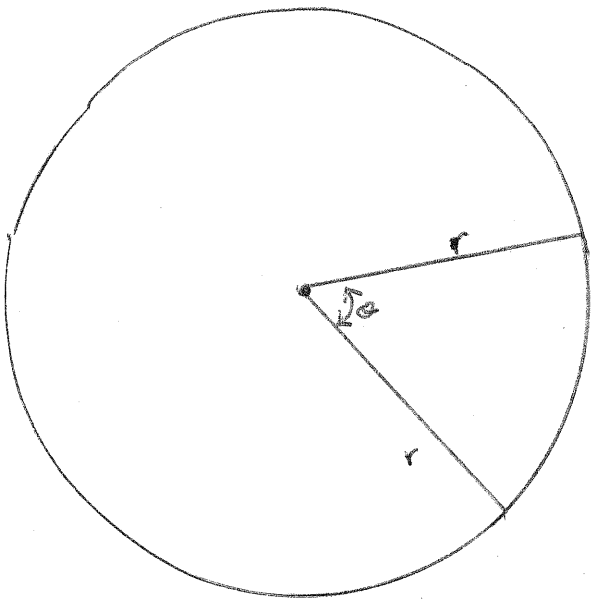
$S = r\theta$
$S = 5 \times 1.4$
$S = 7\text{cm}$

2) Find the arc length if the radius is 12cm and $\theta = 120°$.

$$120° = 2 \times 60° = \frac{2\pi}{3}$$

Arc length $S = 12 \times \frac{2\pi}{3}$
$S = 25.1\text{cm}$

Area of a Sector

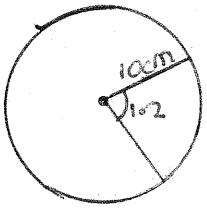


The area of a sector is given by

$A = \frac{1}{2} r^2 \theta$

N.B. θ must be in RADIANS

eg 1)

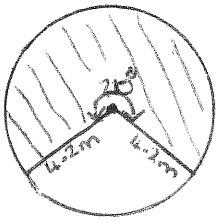


$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2} \times 10^2 \times 1.2$

$A = 60cm^2$

2)



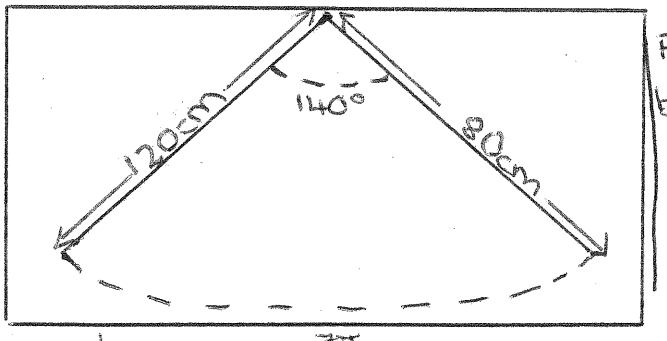
Find the shaded area

$210° = 7 \times 30° = \frac{7\pi}{6}$

$\text{Area} = \frac{1}{2} \times 4.2^2 \times \frac{7\pi}{6}$

$= 32.33m^2$

3)



Find the area of the glass cleaned by the wiper.

$140° = 2.443$ or $\frac{7\pi}{9}$

$A' = \frac{1}{2} \times 120^2 \times \frac{7\pi}{9}$

$A' = 17592.9$

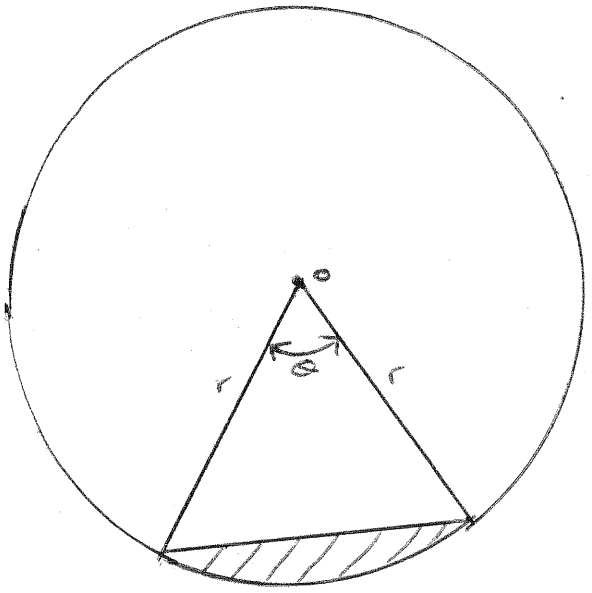
$A_1 = \frac{1}{2} \times 40^2 \times \frac{7\pi}{9}$

$A_2 = 1954.8$

$\text{Area} = A_1 - A_2$

$= 15638.2cm^2$

Area of segment



Find the area of the shaded segment if $r = 8cm$ and $\theta =$

$$A_{sector} = \frac{1}{2} \times 8^2 \times \left(\frac{2\pi}{9}\right)$$
$$= 22.34$$

$$A_{\Delta} = \frac{1}{2} \times 8^2 \times \sin\left(\frac{2\pi}{9}\right)$$
$$= \frac{1}{2} \times 64 \times \sin\left(\frac{2\pi}{9}\right)$$
$$= 20.57$$

$$\text{Area segment} = 22.34 - 20.57$$
$$= 1.8cm^2$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$