Achievement Standard 2.6
Probability

Probabilities can be determined using:
- a simulation
- theoretical probability

A simulation involves an expirament using random numbers to simulate a probability situation.

Theoretically probability often involves calculating probabilities from a probability tree diagram.

Simulation

A group of 20 students are being introduced to the game of basketball. History shows that, after a short practice, students are successful with 30% of their free throws.
Each student is to have 12 free throws.
You are to carry out a simulation to estimate the number of successful throws for each student. Carry out the simulation for all 20 students.

a) Describe a simulation using random numbers to model the situation above.
- Set calculator to generate random integers between 1 and 10
- Let 1 or 2 or 3 to represent a successful throw, and 4-10 represent a unsuccessful throw.
- Generate 12 integers for each student, and record the number of successes for each student.
- Repeat this simulation for all 20 students.

b) Carry out the simulation

	Student 1		Student 2		Student 3		Student 4		Student 5		Studer
1	5	✗	2	✓	8	✗	10	✗	4	✗	9
2	6	✗	10	✗	7	✗	1	✓	10	✗	4
3	8	✗	9	✗	7	✗	2	✓	3	✓	5
4	8	✗	6	✗	4	✗	8	✗	9	✗	7
5	9	✗	7	✗	4	✗	5	✗	2	✓	10
6	7	✗	4	✗	8	✗	4	✗	7	✗	6
7	4	✗	2	✓	1	✓	4	✗	7	✗	5
8	5	✗	4	✗	8	✗	10	✗	7	✗	8
9	7	✗	3	✓	10	✗	6	✗	7	✗	9
10	6	✗	5	✗	6	✗	8	✗	4	✗	7
11	6	✗	1	✓	1	✓	3	✓	10	✗	8
12	1	✓	1	✓	2	✓	4	✗	5	✗	4
		1		5		3		3		2	

	Student 7		Student 8		Student 9		Student 10		Student 11		~~Student 12~~		Stude
1	1	✓	8	✗	2	✓	7	✗	10	✗	~~9~~	~~✗~~	3
2	4	✗	3	✓	10	✗	6	✗	1	✓	~~3~~	~~✓~~	1
3	1	✓	2	✓	1	✓	9	✗	9	✗	~~10~~	~~✗~~	4
4	2	✓	4	✗	1	✓	1	✓	3	✓	~~10~~	~~✗~~	8
5	1	✓	7	✗	8	✗	3	✓	6	✗	~~7~~	~~✗~~	7
6	3	✓	6	✗	4	✗	1	✓	3	✓	~~10~~	~~✗~~	3
7	2	✓	7	✗	6	✗	1	✓	10	✗	~~7~~	~~✗~~	2
8	6	✗	3	✓	7	✗	4	✗	3	✓	~~2~~	~~✓~~	3
9	4	✗	5	✗	6	✗	3	✓	10	✗	~~7~~	~~✗~~	2
10	2	✓	3	✓	3	✓	10	✗	5	✗	~~3~~	~~✓~~	10
11	1	✓	8	✗	2	✓	3	✓	1	✓	~~10~~	~~✗~~	6
12	8	✗	5	✗	3	✓	8	✗	5	✗	~~3~~	~~✓~~	4
		8		4		6		6		5		~~4~~	

Student 10	1	2	3	4	5	6	7	8	9	10	11	12	
	1	3	3	6	2	5	7	10	1	2	7	7	
	✓	✓	✓	✗	✓	✗	✗	✗	✓	✓	✗	✗	6

Student 13		Student 14		Student 15		Student 16		Student 17		Student 18		Student 19	
5	×	5	×	1	✓	9	×	10	×	2	✓	3	✓
2	✓	8	×	3	✓	3	✓	1	✓	2	✓	1	✓
5	×	8	×	3	✓	10	×	2	✓	8	×	9	×
9	×	9	×	10	×	8	×	3	✓	6	×	7	×
8	×	5	×	10	×	2	✓	6	×	5	×	7	×
4	×	7	×	9	×	5	×	1	✓	1	✓	9	×
6	×	2	✓	2	✓	10	×	5	×	4	×	5	×
2	✓	4	×	5	×	10	×	5	×	9	×	6	×
6	×	9	×	6	×	8	×	6	×	7	×	9	×
2	✓	3	✓	3	✓	10	×	7	×	8	×	7	×
9	×	9	×	7	×	2	✓	4	×	9	×	7	×
1	✓	6	×	2	✓	10	×	9	×	10	×	6	×
	4		2		6		3		4		3		2

c) Summarise your results on the table below

Number of successful throws x	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	1	3	4	3	2	5	0	1	0	0	0	0
Probability of x successes	0.05	.05	.15	.2	.15	.1	.25	0	.05	0	0	0	0

d) Based on your simulation, what is the likely number of successful throws a student may have?

6 successes

e) Later a similar group of 50 students were put through the same task of having 12 free throws.
Based on your simulation results:

1) how many students of the 50 would have 2 successful throws?

$$50 \times .15 = 7.5 \text{ students}$$
$$= 8 \text{ students}$$

2) how many of the 50 students would have at least 3 successful throws?

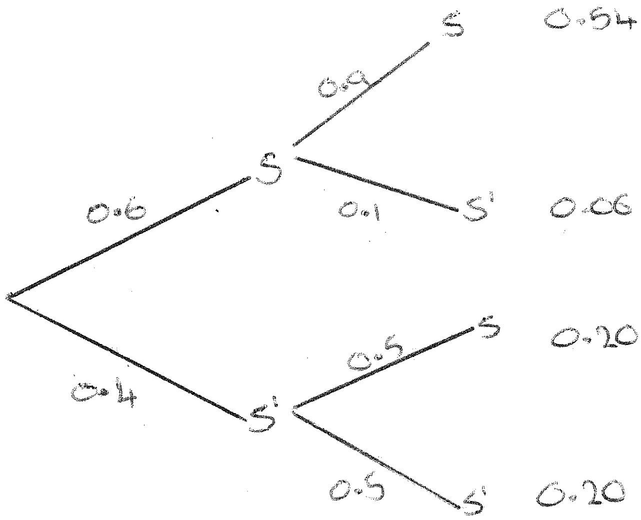
$$50 \times .75 = 37.5 \text{ students}$$
$$= 38 \text{ students}$$

f) In basketball you get 2 free throws. A good player has a probability of 0.6 of his first throw being successful, the probability of the second being successful is 0.9. If the first throw misses, the probability of the second throw being successful is only 0.5.

Use theoretical probability to answer the following:

i) If a player goes to the free throw line on 20 occasions, find how many occasions he has 2 successes.

ii) On how many occasions he has exactly 1 succe



i) P(2 successes) = 0.54

Number occasions = 20 × 0.54
= 10.8
= 11

ii) P(exactly 1 success) = 0.06 + 0.20
= 0.26

Number of occasions = 20 × 0.26
= 5.2
= 5

Normal Distribution

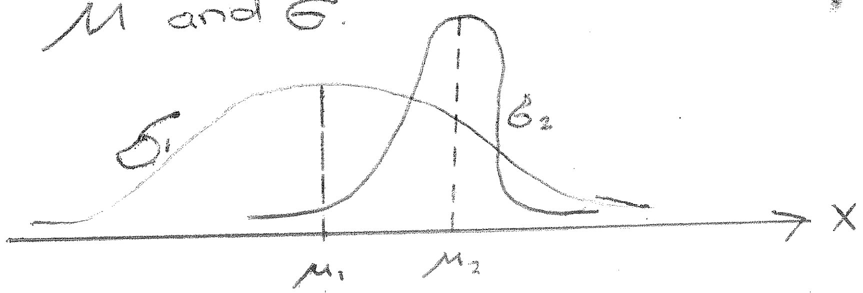
Any populations when graphed have a bell-shaped curve

eg - Weights of new born babies
- Heights of trees in a population
- Lifetimes of lightbulbs
- Lengths of trout in a lake

Such populations are said to be <u>normally distributed</u>

The mean of a population is μ
The standard deviation of a population is σ
(standard deviation is a measure of spread)

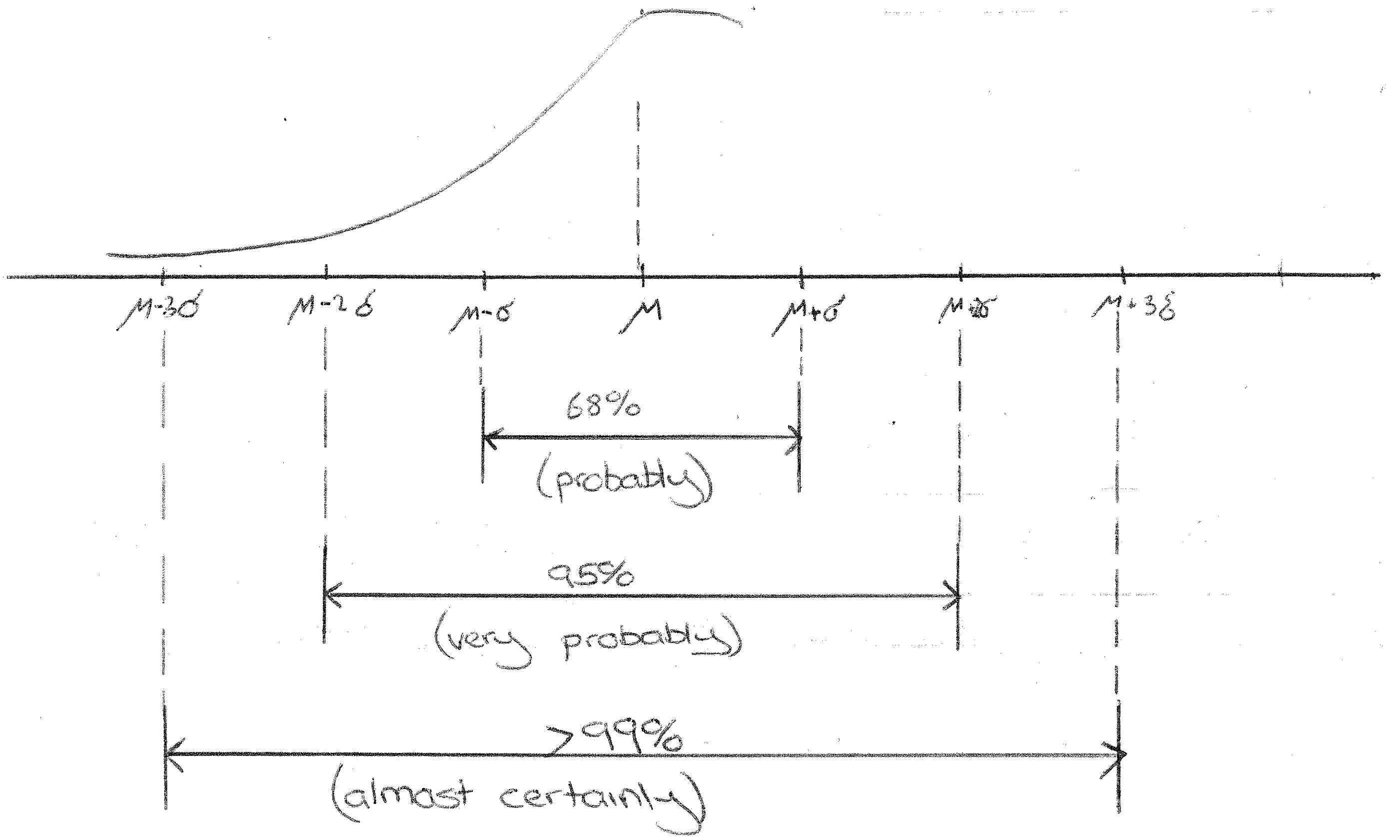
The shape of the normal curve depends on the values of μ and σ.



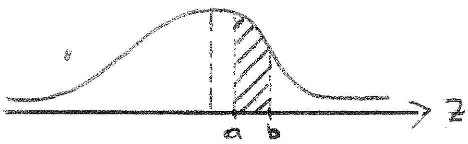
Notice that $\mu_1 < \mu_2$

and $\sigma_1 > \sigma_2$

Any shaped normal curve X can be transferred to a standard normal curve Z, having the following properties:



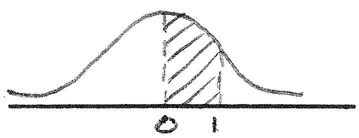
The total area under a standard normal curve is 1.
The probability that Z lies between a + b is the area under the curve between Z = a and Z = b

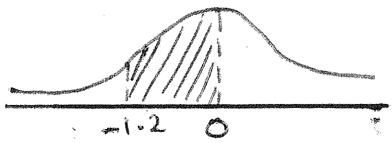


$P(a < Z < b)$ = shaded area

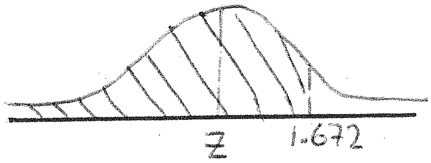
Tables exist for finding areas under standard normal curves

Examples
1) $P(0 < Z < 1) = 0.3413$ 2) $P(-1.24 < Z < 0) = .3925$

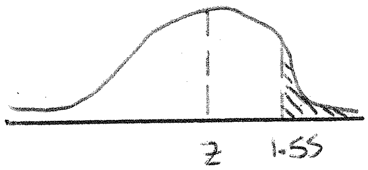




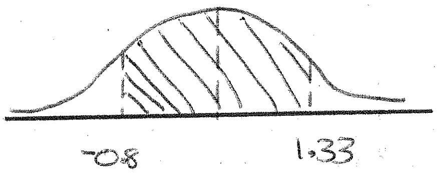
3) $P(Z < 1.672) = 0.5 + 0.4527 = 0.9527$



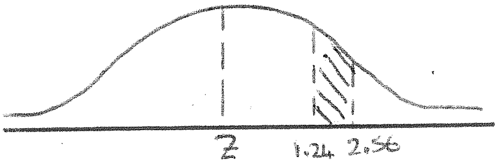
4) $P(Z > 1.55) = 0.5 - 0.4394 = \cancel{0.1606}$
$= 0.0606$



5) $P(-0.8 < Z < 1.33) = 0.2881 + 0.4082 = 0.6963$



6) $P(1.24 < Z < 2.56) = 0.4948 - 0.3925 = 0.1023$



To transform any normal distribution x to a standard normal distribution Z we use the formula

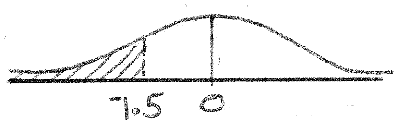
$$Z = \frac{x - \mu}{\sigma}$$

Example

The weights 150 year 12 students are normally distributed with mean $\mu = 65$kg and s.d. $\sigma = 10$kg

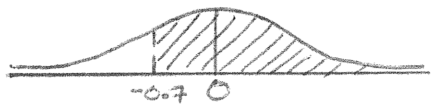
a) Find the probability that a randomly chosen student weighs less than 50kg

$$\begin{aligned} P(x < 50) &= P\left(Z < \frac{50-65}{10}\right) \\ &= P(Z < -1.5) \\ &= 0.5 - 0.4332 \\ &= 0.0668 \end{aligned}$$



b) What proportion of students weigh more than 58kg

$$\begin{aligned} P(x > 58) &= P\left(Z > \frac{58-65}{10}\right) \\ &= P(Z > -0.7) \\ &= 0.2580 + 0.5 \\ &= 0.7580 \end{aligned}$$



c) A prop should weigh between 80kg and 85kg

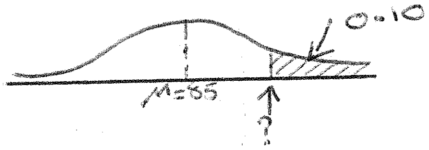
How many of the students are suitable to be props

$$\begin{aligned} P(80 \leq X \leq 85) &= P\left(\frac{80-65}{10} \leq Z \leq \frac{85-65}{10}\right) \\ &= P(1.5 \leq Z \leq 2.0) \\ &= 0.4772 - 0.4332 \\ &= 0.0440 \end{aligned}$$

$$\begin{aligned} \text{Number} &= 150 \times 0.0440 \\ &= 7 \end{aligned}$$

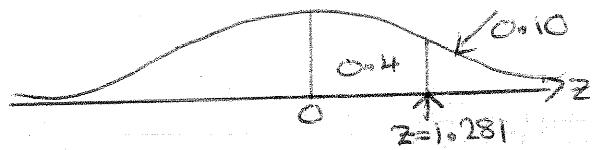
Inverse Normal

1) The weights of N.Z. males are normally distributed with mean $\mu = 85$kg and s.d. $\sigma = 9$kg
10% of N.Z. males are fat
What weight is required before you are classed as fat.



Let the weight be k

$$P(X > k) = 0.10$$
$$P\left(Z > \frac{k-85}{9}\right) = 0.10$$



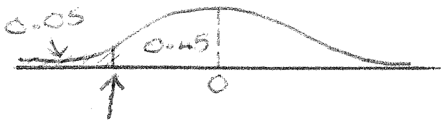
$$\frac{k-85}{9} = 1.281$$

$$k - 85 = 11.5$$
$$k = 96.5\text{kg}$$

2) The life-times of TV sets are normally distributed with mean $\mu = 6$years and s.d $\sigma = 1.2$years
The manufacturer is prepared to replace 5% of TV sets under guarantee
What guarantee period should he give?

Let guarantee period be k

$$P(X < k) = 0.05$$
$$P\left(Z < \frac{k-6}{1.2}\right) = 0.05$$



$z = -1.645$

$$\frac{k-6}{1.2} = -1.645$$

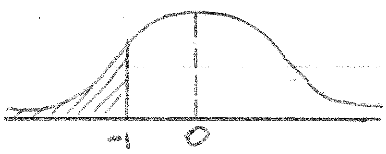
$$k - 6 = {}^{-}2.0$$

$$k = 4 \text{ years}$$

3) A manufacturer bags and sells flour in 500g bags. He sets his machinery to dispense flour with a mean $\mu = 510$g. The s.d. is known to be 10g.

a) What proportion of bags are underweight?

$$P(X < 500) = P\left(Z < \frac{500-510}{10}\right)$$

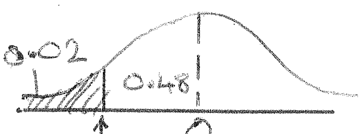


$$= P(Z < -1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

b) Concerned about this, the manufacturer re-sets his machine. What mean should he set it to so that only 2% of bags are underweight?

$$P(X < 500) = 0.02$$
$$P\left(Z < \frac{500-\mu}{10}\right) = 0.02$$

He should set his machine to a mean of 520.5



$z = {}^{-}2.054$

$$\frac{500-\mu}{10} = {}^{-}2.054$$

$$500 - \mu = -20.54$$

$$-\mu = -520.54$$

$$\mu = 520.5$$

Graphics Calculator

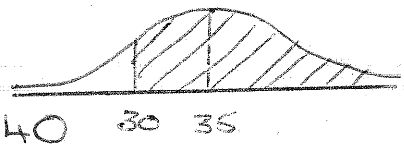
You must state the parameters of the distribution (μ and σ
You must write a probability statement

Menu - STAT - DIST - NORM - Ncd to find a probability (A, M)
InvN to do an "inverse normal problem. (E)

Eg The lengths of T.V. advertisements are normally distributed with mean $\mu = 35$ seconds and s.d. $\sigma = 8$ seconds.

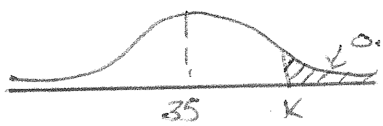
a) What proportion of ads last more than 30 seconds?

$\mu = 35$
$\sigma = 8$
$P(X > 30) = 0.7340$



b) Find the length of ad above which only 10% h

$\mu = 35$
$\sigma = 8$
$P(X > k) = 0.10$
$k = 45.3$ seconds



Area always fro
left $= 0.9$