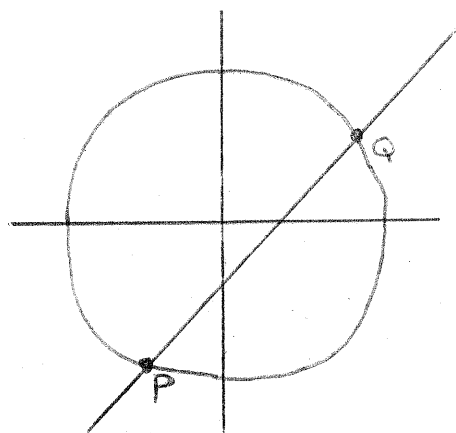


## Intersection of a line and curve

- 1) Find the points of intersection of the circle  $x^2 + y^2 = 74$  and the line  $y = 2x - 3$

$$\begin{aligned}x^2 + (2x - 3)^2 &= 74 \\x^2 + 4x^2 - 12x + 9 &= 74 \\5x^2 - 12x - 65 &= 0\end{aligned}$$

$$x \in \{5, -2.6\}$$

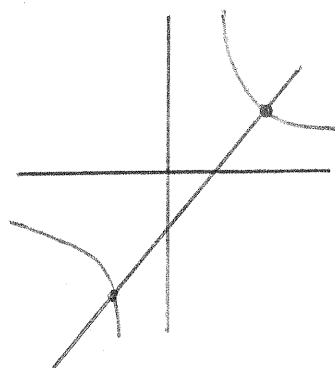


When  $x = 5$   
 $y = 7$

$x = -2.6$   
 $y = 2(-2.6) - 3$   
 $y = -8.2$

P  $(-1.6, -8.2)$  and Q  $(5, 7)$

- 2) Find the intersection of the hyperbola  $xy = 24$  and the line  $y = 3x - 1$



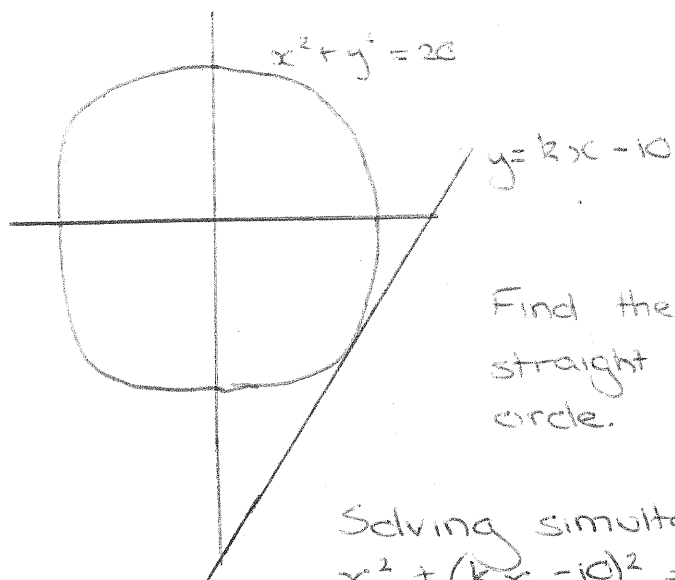
$$\begin{aligned}x(3x - 1) &= 24 \\3x^2 - x &= 24 \\3x^2 - x - 24 &= 0 \\x &\in \{3, 2\frac{2}{3}\}\end{aligned}$$

When  $x = 3$   
 $y = 8$

When  $x = -2\frac{2}{3}$   
 $y = 3(-2\frac{2}{3}) - 1$   
 $y = -9$

Intersect at  $(3, 8)$  and  $(-2\frac{2}{3}, -9)$

3)



Find the value of  $k$  so that the straight line is a tangent to the circle.

Solving simultaneously,

$$x^2 + (kx - 10)^2 = 20$$

$$x^2 + k^2 x^2 - 20kx + 100 = 20$$

$$x^2 + k^2 x^2 - 20kx + 80 = 0$$

$$(k^2 + 1)x^2 - 20kx + 80 = 0$$

Require  $b^2 - 4ac = 0$

$$(-20k)^2 - 4(k^2 + 1) \cdot 80 = 0$$

$$400k^2 - 320(k^2 + 1) = 0$$

$$80k^2 = 320$$

$$k^2 = 4$$

$$k = \pm 2$$