Linear Programming

Linear programming involves maximising a profit or minimising your costs subject to a set of linear constraints

Examples:

(1) A school is making tee shirts & singlets to raise funds.

	Number made	Material required	Time to make each item	Profit per item
Tee-shirts	x	1·4m	1hr	12
Singlets	y	0·7m	1·5hr	8

There are 112m of material available, & 120 hours of labour available. The school already has orders for 15 tee-shirts & 20 singlets. Find how many of each item they should make in order to maximise their profit. Find this profit

Step 1:

Write your constraints:

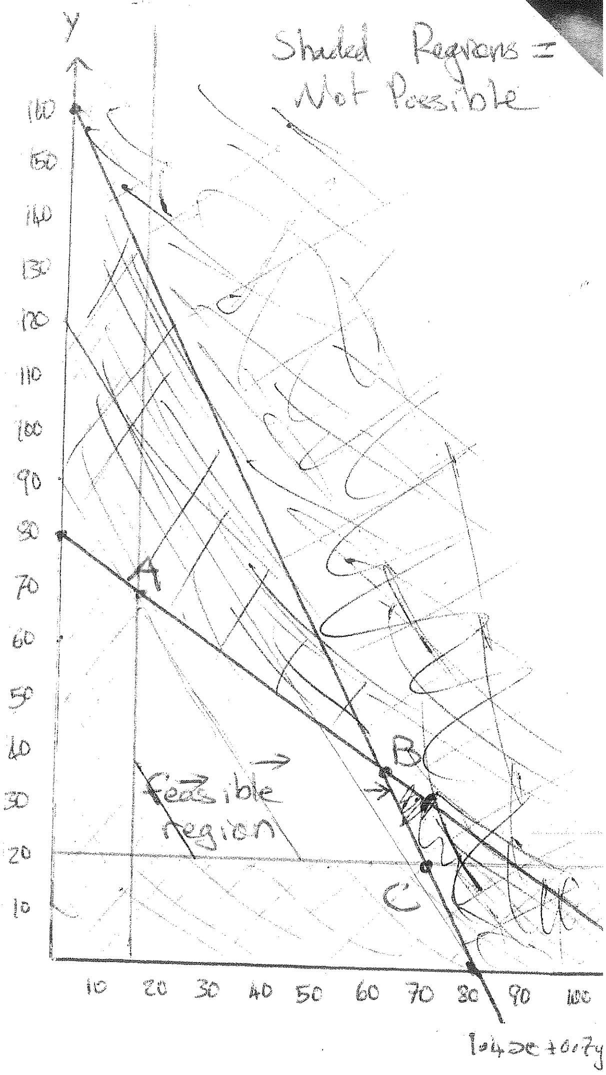
$x \geq 15$

$y \geq 20$

$1{\cdot}4x + 0{\cdot}7y \leq 112$

$x + 1{\cdot}5y \leq 120$

Step 2: Graph the constraints & locate the feasible region



Step 3: Profit $P = 12x + 8y$

Step 4: Rearrange profit equation to make y the subject.

$P = 12x + 8y$

$8y = -12x + P$

$y = \frac{-12}{8}x + \frac{P}{8}$

$y = -\frac{3}{2}x + \frac{P}{8}$

Slope of profit line $= \frac{-3}{2}$

Max. profit occurs at B(60,40)

We should make 60 tee-shirts & 40 singlets.

$P_{max} = 12(60) + 8(40)$
$= \$1040$

(2) An oil company operates two refineries. These produce three types of fuel: aviation grade, regular grade petrol and super grade petrol.

The first refinery costs \$160 000 a day to operate, and the second refinery costs \$175 000 a day to operate.

The oil company has contracts to produce at least 120 000 litres of aviation fuel, 300 000 litres of regular grade petrol and 108 000 litres of super grade petrol per month.

This table gives the daily production statistics in litres:

	Number of days per month	Quantity of aviation fuel	Quantity of regular petrol	Quantity of super petrol
Refinery 1	x	10 000	20 000	6000
Refinery 2	y	10 000	30 000	18 000

Determine how many days per month each refinery should operate in order to minimise their costs.

$5x + 1.5y \le 120$

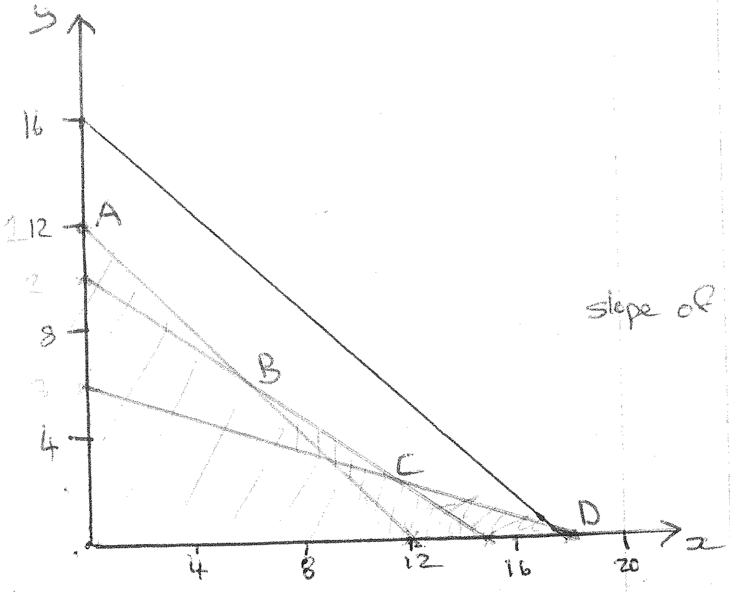
Constraints are: $x \geq 0$

$y \geq 0$

$10000x + 10000y \geq 120\,000$ (aviation fuel)

$20000x + 30\,000y \geq 300\,000$ (regular petrol)

$6000x + 18000y \geq 108\,000$ (Super petrol)



Cost equation:

$C = 160000x + 175000y$

$y = \dfrac{C - 160000x}{175000}$

$= -\dfrac{32}{35} + \dfrac{C}{175000}$

slope of cost line is $-\dfrac{32}{35}$ $\dfrac{-16}{17.5}$

$\therefore$ B is where min occurs

as $10000x + 10000y \geq 12000$ $m = -1$

slope is less than -1 so $\therefore$ B.

$$2x + 2y = 24$$

$$y = 6$$

$$x = 6$$

refinery 1 & refinery 2 should both operate for 6 days per month

$$C_{min} = 160000(6) + 175000(6)$$

$$= \$2\,010\,000$$

QUESTION THREE

(a) A newspaper recycling company has facilities to produce toilet rolls and writing pads. Toilet rolls consume 0.5 kg of newspaper whilst writing pads consume 0.2 kg. It takes 0.2 minutes to produce a toilet roll and 0.4 minutes to produce a writing pad. Each day 8 hours and 800 kg of newspaper are available. Each day orders are such that at least 1000 rolls of toilet paper are required and at least 200 writing pads. The profit on a toilet roll is 18 cents and on a writing pad 30 cents.

(i) Determine the maximum total profit that can be obtained, and the production of toilet rolls and writing pads that would generate this profit.

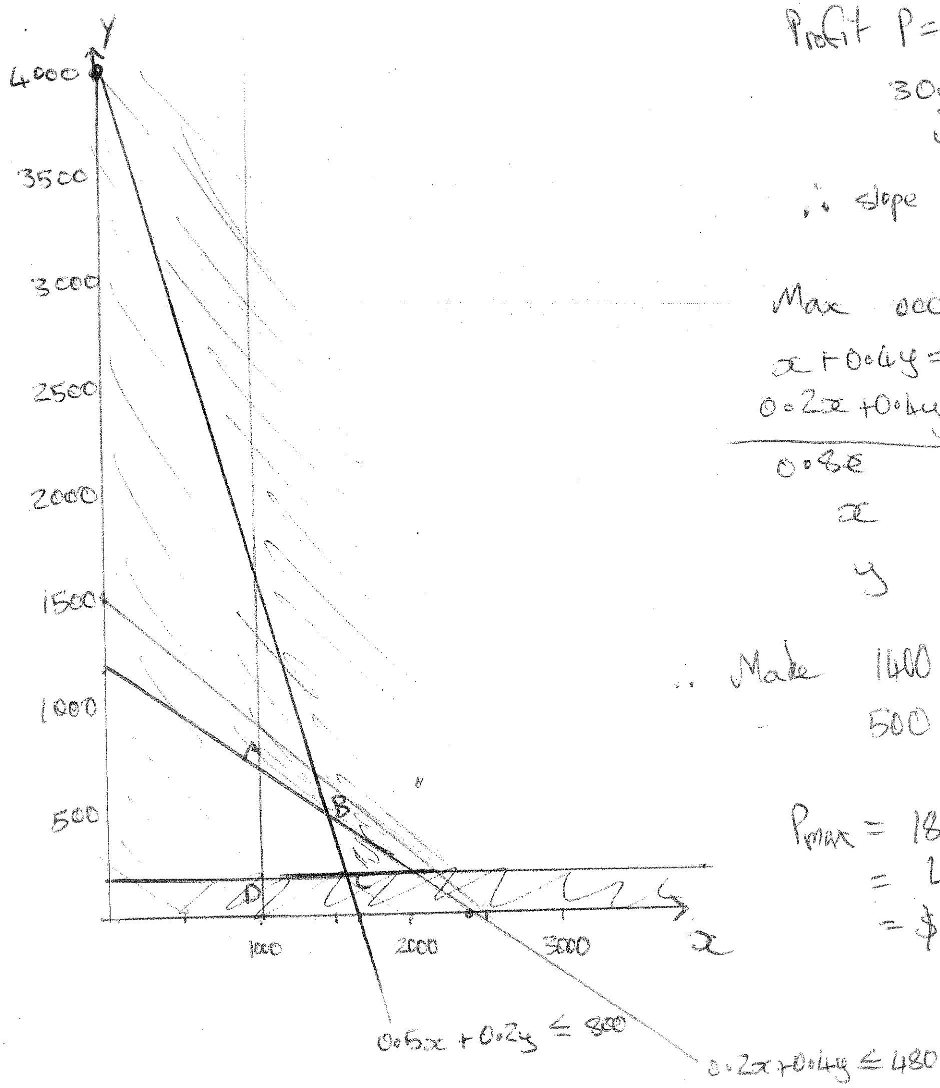
(ii) If, instead, the profit on a toilet roll is 15 cents and on a writing pad 30 cents, what is the maximum total profit that can be obtained? Determine all possible production orders that will generate this revenue.

(iii) Orders are subsequently received that require at least 1600 toilet rolls per day, with the number of writing pads remaining as at least 200 per day. What options does the company have with respect to this new order? Give concise explanations.

Let number of toilet rolls be x
number of writing pads be y

$x \geq 1000$
$y \geq 200$

$0.5x + 0.2y \leq 800$
$0.2x + 0.4y \leq 480$



Profit $P = 18x + 30y$

$30y = -18x + P$

$y = -\frac{3}{5}x + \frac{P}{30}$

$\therefore$ slope profit line $= -\frac{3}{5}$

Max occurs at B

$x + 0.4y = 1600$
$0.2x + 0.4y = 480$

$0.8x = 1120$
$x = 1400$
$y = 500$

$\therefore$ Make 1400 toilet paper rolls
500 writing pads

$P_{max} = 18(1400) + 30(500)$
$= 40200¢$
$= \$402$

ii) $P = 15x + 30y$

$30y = -15x + P$

$y = -\frac{1}{2}x + \frac{P}{30}$

slope profit line $= -\frac{1}{2}$

This is the same as the slope of line AB, whose equation is $x + 2y = 2400$

A is $(1000, 700)$
B is $(1400, 500)$

Solution is

x	y
1000	700
1002	699
1004	698
1006	697
⋮	⋮
1400	500

iii) There is now no feasible region.
The constraint for newspaper $(0.5x + 0.2y = 800)$ must move to point D to form a feasible region.
D is $(1600, 200)$

The new equation for newsprint constraint is
$0.5x + 0.2y \geq \underline{\underline{840}}$

Thus the company has two options: either decline the extra orders or increase the amount of newspaper available per day to 840kg.