Achievement Standard 3.3 - Probability - 4 credits

Permutations & Combinations

Permutations are arangements (order matters)

Combinations are selections (order does not matter).

Permutations

The permutations of the letters A, B & C are
ABC, ACB, BAC, BCA, CAB, CBA
ie 6 different positions.
We could have arrived at this using the Multiplication Rule

1st	2nd	3rd
3	2	1

$= 3 \times 2 \times 1 = 6$ permutations

3 choices, 2 choices, 1 choice

Examples (1): We have 6 boys to choose from. How many ways can they be arranged in a row if I choose

(a) 2 of them

| 6 | 5 | = 30

(b) 4 of them

| 6 | 5 | 4 | 3 | = 360

(2) How many <u>even</u> 3 digit numbers can be **formed** from the digits 1, 2, 5, 6, 7, 9 if they can be used only once.

| 5 | 4 | 2 | = 40 numbers

fill this position first only 2 or 6 can go there.

(3) A number plate consists of 3 letters followed by up to 3 digits.

How many number plates are possible?

$\boxed{26 \mid 26 \mid 26 \mid 999} = 17558424$

The number of permutations of 'n' distinct objects arranged in a line is $n!$ (n factorial)

eg(1) How many ways can 5 people be arranged in a row?
$5! = 120$

(2) Mum & Dad & 3 children line up for the bus. How many arangements are possible in which Mum & Dad are not together?

Number of arrangements $= 5! = 120$
Number of arrangements with mum & dad together $= 4! \times 2 = 48$
$\therefore$ Number of arrangements when NOT together $= 120 - 48$
$= 72$

The number of permutations of 'n' distinct objects chosen 'r' at a time is $^nP_r = \frac{n!}{(n-r)!}$

eg(1) There are 8 horses in a race. How many trifectas would you have to take to cover every possibility?

$^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6$
$= 336$ trifectas.

Combinations

The symbol ${}^nC_r$ denotes the number of combinatio of n distinct objects chosen r at a time.

$${}^nP_r = {}^nC_r \times r!$$

$$\therefore \quad {}^nC_r = \frac{{}^nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

Example: (1) Evaluate

$$ {}^8C_3 = \frac{8!}{(8!-3!)\,3!}$$
$$= \frac{8\times7\times6\times5\times4\times3\times2\times1}{5!\ 3!}$$
$$= \frac{8\times7\times6\times\cancel{5\times4\times3\times2\times1}}{\cancel{5\times4\times3\times2\times1}\times3\times2\times1}$$
$$= \frac{8\times7\times6}{3\times2\times1}$$
$$= 56$$

(2) How many ways can a committee of 3 people be chosen from 10 people?

${}^{10}C_3 = 120$ committees

(3) A committee of 5 is to be choosen from 6 men & 8 women.
What is the probability that the comittee consists of 2 men & 3 women

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{{}^6C_2 \times {}^8C_3}{{}^{14}C_5}$$

$$= 0.4196$$

Basic Probability

The set of all possible outcomes of a statistical expt. (experiment) is called the Sample Space S

eg (1) Rolling a dice.
$S = \{1, 2, 3, 4, 5, 6\}$

2) Toss two coins
$S = \{HH, HT, TH, TT\}$

(3) Have 3 Babies
$S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$

An event is a subset of the sample space. Suppose event A is "2 of the 3 babies are girls" (Question 3)

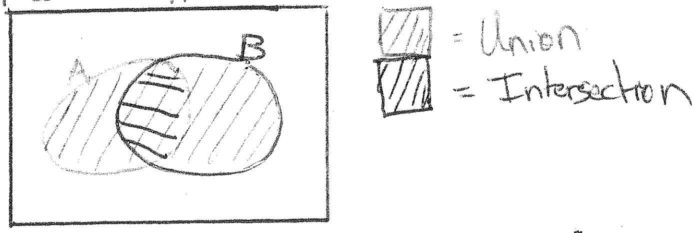
P = Probability
$P(\text{event A occurring}) = 3/8$
or $P(A) = 3/8$

Suppose we select a card at random from a pack of 52 cards.
a) $P(\text{Heart Chosen}) = 13/52 = 1/4$
b) $P(\text{King Chosen}) = 4/52 = 1/13$

Additive Rule

If A and B are two events, then the probability of A occurring OR B occurring corresponds in a Venn Diagram to $P(A \cup B)$

$\cup$ means "union" of 2 sets.
$\cap$ means "intersection" of 2 sets



$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Using our pack of 52 cards,

$$P(\text{Heart or King}) = P(\text{heart}) + P(\text{king}) - P(\text{heart} \cap \text{king})$$

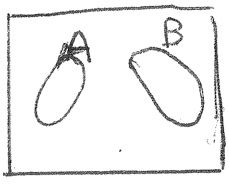
$$= \left(\tfrac{13}{52}\right) + \left(\tfrac{4}{52}\right) - \left(\tfrac{1}{52}\right)$$

$$= \tfrac{16}{52}$$

$$= \tfrac{4}{13}$$

Mutally Exclusive Events

Events A & B are mutually exclusive if they do not ov on a Venn Diagram



In this case $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

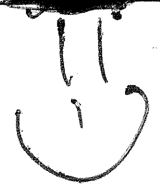
Complementary Events

$P(A) = 0{\cdot}4$
$P(A') = 0{\cdot}6$

Suppose A is an event.
Then "not A" is the complementary event.
It is written A'

$$P(A) + P(A') = 1$$

[redacted]



Sample Problem

The probability of a student passing Maths is $\frac{2}{3}$, and the probability of passing English is $\frac{2}{5}$.
The probability of passing at least one of these subjects is $\frac{9}{10}$
Find the probability of passing both subjects.

$P(M) = \frac{2}{3}$
$P(E) = \frac{2}{5}$
$P(M \cup E) = \frac{9}{10}$

Require: $P(M \cap E)$

$$P(M \cup E) = P(M) + P(E) - P(M \cap E)$$
$$\frac{9}{10} = \frac{2}{3} + \frac{2}{5} - P(M \cap E)$$
$$P(M \cap E) = \frac{2}{3} + \frac{2}{5} - \frac{9}{10}$$
$$= \frac{20 + 12 - 27}{30}$$
$$= \frac{5}{30} \text{ or } \frac{1}{6}$$

Independent Events

If A & B are 2 events then the probability of A occurring $\boxed{\text{and}}$ B occurring corresponds on a Venn Dia to P(A∩B).

If A & B are independent events, then:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

Examples (1) A dice is rolled & a coin is tossed

Find P(six & Head)

$= P(\text{six}) \times P(H)$

$= \frac{1}{6} \times \frac{1}{2}$

$= \frac{1}{12}$

(2) A dice is rolled 3 times. Find the probabilit of getting no sixes.

$P(\text{no sixes}) = P(6' \,\&\, 6' \,\&\, 6')$

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{125}{216}$

(3) Simon went camping. He took 10 tins of food, of which had lost their labels.

He knows that 6 contain peaches & 4 contain baked beans.

For breakfast he opens 3 tins.

What is the probability they are all peaches?

$P(\text{all peaches}) = P(P \,\&\, P \,\&\, P)$

$= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$

$= \frac{1}{6}$

4) If the Independant probability that A, B & C will be alive in 20 years time are 0.4, 0.3 & 0.2 respectively, find the probability that in 20 years time:

a) All are alive
b) None are alive
c) One is alive
d) At least one is alive

a) $P(\text{all alive}) = P(A \;\&\; B \;\&\; C)$
$= 0.4 \times 0.3 \times 0.2$
$= 0.024$

b) $P(\text{none alive}) = P(A' \;\&\; B' \;\&\; C')$
$= 0.6 \times 0.7 \times 0.8$
$= 0.336$

c) $P(\text{one is alive}) = P(AB'C' \text{ or } A'BC' \text{ or } A'B'C)$
$= 0.4 \times 0.7 \times 0.8 + 0.6 \times 0.3 \times 0.8 + 0.6 \times 0.7 \times 0.2$
$= 0.452$

d) $P(\text{at least one alive}) = 1 - P(\text{none alive})$
$= 1 - 0.336$
$= 0.664$

5) OVER PAGE

5) The table shows sex and employment status of 1000 adults in a town.

	Employed	Unemployed
Male	500	100
Female	200	200

Are the events "being male" & "being employed" independant e

From the Table:

$$P(M) = \frac{600}{1000} = 0.6$$
$$P(E) = \frac{700}{1000} = 0.7$$
$$P(M \cap E) = \frac{500}{1000} = 0.5$$

Independent if $P(M \cap E) = P(M) \times P(E)$

$$0.5 \neq 0.6 \times 0.7$$

$\therefore$ events are $\boxed{\text{not}}$ independent.

Conditional Probability

The probability of event A occurring when it is known that event B has occurred is called a conditional probability, & is denoted by $P(A/B)$, and read as, "the probability of A given B".

eg

	Employed	Unemployed
Male	500	100
Female	200	200

A person is chosen at random.

Given that an employed person was chosen, what is the probability that a male was chosen?

We require $P(M/E)$
The condition imposed reduces the sample space from 1000 to the 700 employed people.
Thus $P(M/E) = \frac{500}{700} = \frac{5}{7}$

Note $P(M/E) = \frac{500}{700} = \frac{500/1000}{700/1000} = \frac{P(M \cap E)}{P(E)}$

Thus $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Examples:

1) The probability that a plane on a regular flight departs on time is
$P(D) = 0.83$, & the probability of arriving on time is
$P(A) = 0.92$, & the probability that it departs & arrives on time is
$P(D \cap A) = 0.78$
Find the probabilities that:
a) It arrives on time given that it departed on time
$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0.78}{0.83} = 0.94$

b) It departed on time given that it arrives on time

$$P(D/A) = \frac{P(D \cap A)}{P(A)} = \frac{0.78}{0.92} = 0.85.$$

(2) The independant probabilities that A & B will be alive in 20 years time are 0.7 & 0.4 respectively.

What is the probability that, if just one of them is alive in 20 years time it is B?

We require $P(B / \text{one alive})$.

Now $$P(B / \text{one alive}) = \frac{P(B \cap \text{one alive})}{P(\text{one alive})}$$

$$= \frac{P(B \text{ and } A')}{P(AB' \text{ or } A'B)}$$

$$= \frac{0.4 \times 0.3}{0.7 \times 0.6 + 0.3 \times 0.4}$$

$$= \frac{0.12}{0.54}$$

$$= 2/9$$

Strategies for Probability

The following approaches can be useful.

(1) Write out the sample space.
(2) Draw a Venn Diagram
(3) Draw a Tree Diagram

Examples: (1) Two dice are rolled & the scores added.
What is the probability that the total score is at least 8 given that we know that a "6" was rolled

The best approach here is to write out the sample space.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	$\boxed{9}$
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	$\boxed{9}$	10	11	12

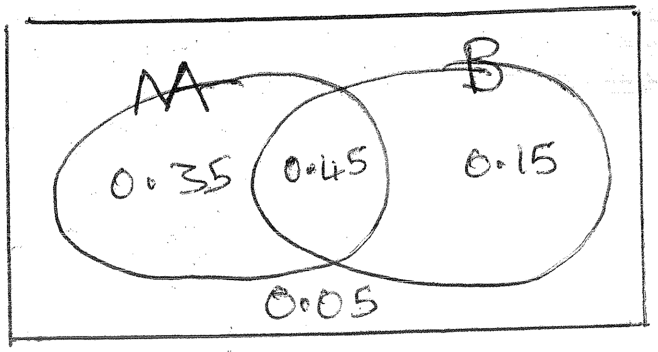
The condition, a 6 was rolled, reduces the sample space to just 11 outcomes
$P(\text{at least } 8 / 6 \text{ occured}) = 9/11$

Calculate the probability that the other dice shows a 3.
$P(3 / 6 \text{ occured}) = 2/11$

2) A school heating plant has a main system & a backup system.
Both are subject to random breakdowns.
The main system M is operational 80% of the time, & at least one system is operational 95% of the time. Both systems are operational for just 45% of the time.

Given that the backup system B is not operati
find the probability that the main system M is oper

The best approach is to draw a Venn Diagram

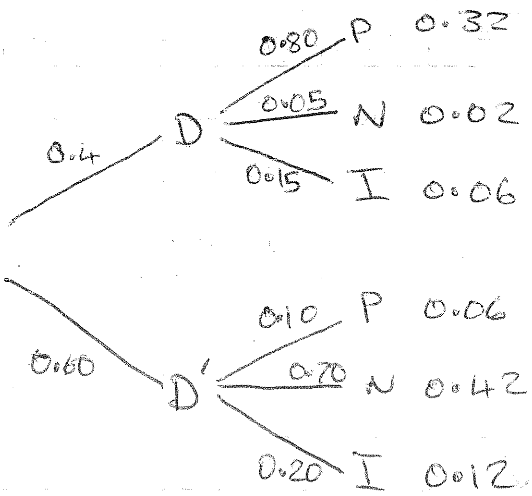


$$P(M/B') = \frac{P(M \cap B')}{P(B')}$$

$$= \frac{0.35}{0.44}$$

$$= 7/8$$

(3) Medical studies on a new disease are being undertaken.
A test has been developed to detect the pressence of the disease.
Suppose that 40% of patients actually have the disease.
Of these, when tested, 80% give a positive return, 5% give a negative return, & the remaining 15% are incbnclusive.
The corresponding figures for those who do not have the disease are 10%, 70% & 20% respectively.



a) Find the probability that a randomly chosen patient gives an inconclusive test result.

$$P(I) = 0.06 + 0.12$$
$$= 0.18$$

b) Given that the test result is positive, find the probability that the patient actually has the disease.

$$P(D/P) = \frac{P(D \cap P)}{P(P)} = \frac{0.32}{0.38} = 16/19$$

Discrete Random Variables

The different data values x that result from a statistical expt (experiment). may be thought of as the different values that a random variabl X can assume

eg (1) 10 seeds are planted
The random variable X represents the number that germinate.

$x \in \{0, 1, 2, 3, 4 \ldots\ldots 10\}$

(The probability distribution of X is binomial)

(2) the random variable X representing the number of telephone calls I receive in a 1 hour period can take values $x \in \{0, 1, 2, 3 \ldots\ldots\}$

(The probability distribution of X is poisson).

(3) Suppose we toss 3 coins & X represents the nu of heads that occur.
Then X can assume values $0, 1, 2, 3$

Probability Distribution

(1) Suppose we roll a dice.
Let X be the number roll.

A table showing the values of X, & their respective probabilities is called a probability distribution

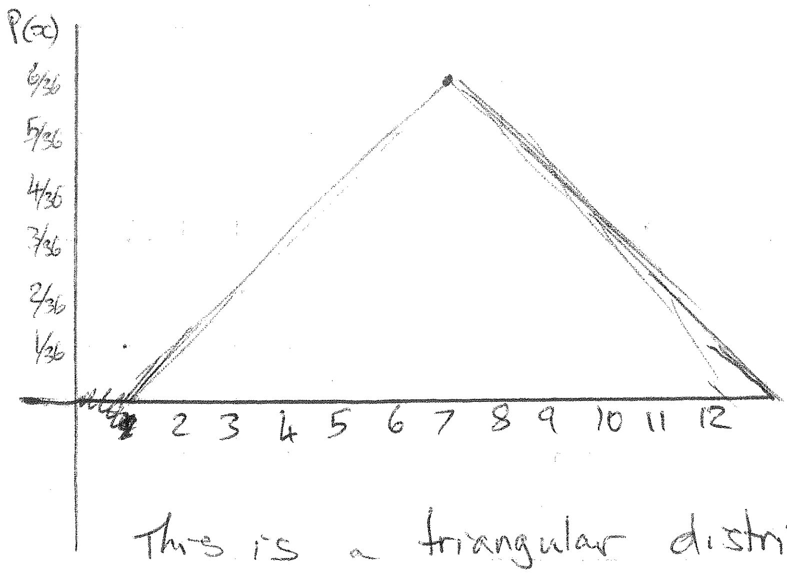
x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

= Uniform Distribution
(all the same)

$$p(x) = P(X = x)$$

(2) Roll 2 dice & add the scores

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



This is a triangular distribution

Mean & Variance

Consider a random variable X whose probability distribution

x	x_1	x_2	x_3	x_4	......	x_n
$p(x)$	$p(x_1)$	$p(x_2)$	$p(x_3)$	$p(x_4)$		$p(x_n)$

The mean μ or expected value $E(X)$ of this random variable is given by

$$\mu_x \text{ or } E(x) = \sum x.p(x)$$

(1) let X be the amount you win in a raffle.

x	0	10	100
$p(x)$	0.7	0.2	0.1

Calculate the expected winnings per ticket

$E(X) = 0 \times 0.7 + 10 \times 0.2 + 100 \times 0.1$
$E(X) = \$12$

(2) Find the average score when a dice is tossed

x	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

$\mu = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$

or $= \frac{21}{6}$

Ex

$= 3.5$

(3) In a gambling game a man is paid \$5 if he gets all heads or all tails when 3 coins are tossed, otherwise he loses \$3.
What is his expected gain per game?

x	5	-3
$p(x)$	$2/8$	$6/8$

$$\mu_x = E(x) = 5 \times 2/8 + -3 \times 6/8$$
$$= -1$$

on average you lose \$1 per game.

A racing car driver wants to insure his car for the season for \$50 000.
The probability of a total loss is 0.002, of an accident involving a 50% loss is 0.01, & a 25% loss has probability 0.1.
What premium should the insurance company charge each season if they want an average profit of \$500 on the policy.

(1) Let the premium be p.
Let x be the profit to the company.

x	p	$p-12500$	$p-25000$	$p-50\,000$
$p(x)$	0.888	0.1	0.01	0.02

$$E(x) = 0.888p + 0.1(p-25000) + 0.01(p-25000) + 0.002(p-500$$
$$= 0.888p + 0.1p - 1250 + 0.01p - 250 + 0.002p - 100$$
$$E(x) = p - 1600$$
Require $E(x) = 500$
$$500 = p - 1600$$
$$p = \$2100$$

Beter Way

(2) Let X be amount insurance company pays out

x	0	12500	25000	50 000
$p(x)$	0.888	0.1	0.01	0.002

$$E(x) = 0 \times 0.888 + 12500 \times 0.1 + 25000 \times 0.01 + 50000 \times 0.002$$
$$= 1600$$

The insurance company can expect to pay out \$1600
∴ premium should be \$2100.

$E(aX+b) = aE(X)+b$

eg consider the probability distribution

x	1	2	3
$p(x)$	0·1	0·8	0·1

Find
(a) $E(X)$
(b) $E(4X)$
(c) $E(X+3)$
(d) $E(6X-7)$
(e) $E(X^2)$

a) $E(X) = 1\times 0{\cdot}1 + 2\times 0{\cdot}8 + 3\times 0{\cdot}1$
$E(x) = 2$

b) $E(4X) = 4\,E(X) = 8$

c) $E(X+3) = E(X)+3 = 5$

d) $E(6X-7) = 6E(X)-7 = 6\times(2)-7 = 5$

e) $E(X^2) = \sum x^2 \times p(x)$
$= 1^2\times 0{\cdot}1 + 2^2\times 0{\cdot}8 + 3^2\times 0{\cdot}1$
$= 0{\cdot}1 + 3{\cdot}2 + 0{\cdot}9$
$= 4{\cdot}2$

(1) Let X be the amount you can win per ticket in a pub raffle

x	0	5	10	20
$p(x)$	0.7	0.15	0.1	0.05

$E(X) = 0 + 5 \times 0.15 + 10 \times 0.1 + 20 \times 0.05$
$E(X) = \$2.75$

For the last raffle of the night the prizes are tripled. Find the expected return per ticket.

We require $E(3X) = 3E(X)$
$= 3(\$2.75)$
$= \$8.25$

(2) You must hire a trailer for a discrete number of hours.

x	1	2	3	4
$p(x)$	0.2	0.4	0.3	0.1

a) Find the average number of hours a trailer is hired

$\mu_X = 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.1$
$= 2.3$ hours

b) The company uses the formular $C = 10X + 5$ to calculate the cost of the hire.
calculate the average cost of a hire

$\mu_{10X+5} = E(10X+5)$
$= 10E(X) + 5$
$= 10(2.3) + 5$
$= \$28.$

Variance

Varience = (standard deviation)^2

It is a measure of the "spread" of a set of data

The variance of a set of data X is written σ_x^2 or VAR(X) ($X\sigma_n$) → on casio (etc)

The variance of X is given by the formula

$$\sigma_x^2 = E[(x-\mu)^2]$$
$$= \sum (x-\mu)^2 \times p(x)$$

Example: Find the s.d. of the random variable x representing the outcome when a dice is rolled

x	1	2	3	4	5	6
$p(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

Recall $\mu = 3{\cdot}5$

$$\sigma_x^2 = VAR(X) = \sum (x-\mu)^2 . p(x)$$
$$= (1-3{\cdot}5)^2 \times 1/6 + (2-3{\cdot}5)^2 \times 1/6 + (3-3{\cdot}5)^2 \times 1/6 + (4-3{\cdot}5)^2 \times 1/6 + (5-3{\cdot}5)^2 \times 1/6 + (6-3{\cdot}5)^2 \times 1/6$$
$$\approx 2{\cdot}9$$
$$\sigma_x = \sqrt{2{\cdot}9}$$
$$= 1{\cdot}7$$

There is a useful alternative formula for finding σ_x^2 or VAR(x)

$$
\begin{aligned}
\sigma_x^2 &= \sum (x-\mu)^2 . p(x) \\
&= \sum (x^2 - 2\mu x + \mu^2) . p(x) \\
&= \sum x^2 . p(x) - \sum 2\mu x . p(x) + \sum \mu^2 . p(x) \\
&= E(X^2) - 2\mu \sum x . p(x) + \mu^2 \sum p(x) \\
&= E(X^2) - 2\mu . \mu + \mu^2 \times 1 \\
&= E(X^2) - 2\mu^2 + \mu^2 \\
&= E(X^2) - \mu^2
\end{aligned}
$$

$$\sigma_x^2 = E(X^2) - [E(x)]^2$$

Example:

x	10	20	30	40
$p(x)$	0.1	0.2	0.4	0.3

Calculate the mean & s.d. of X.

$$
\begin{aligned}
E(x) &= 10 \times 0.1 + 20 \times 0.2 + 30 \times 0.4 + 40 \times 0.3 \\
&\quad 1 + 4 + 12 + 12 \\
&= 29
\end{aligned}
$$

$$
\begin{aligned}
E(X^2) &= 10^2 \times 0.1 + 20^2 \times 0.2 + 30^2 \times 0.4 + 40^2 \times 0.3 \\
&= 10 + 80 + 360 + 480 \\
&= 930
\end{aligned}
$$

$$
\begin{aligned}
\sigma_x^2 &= E(X^2) - [E(X)]^2 \\
&= 930 - 29^2 \\
&= 89
\end{aligned}
$$

ie varience = 89

$\sigma_x = \sqrt{89}$

$= 9.43$

Suppose a random variable X is transformed using $ax + b$.

Recall: $E(ax+b) = aE(x) + b$

$$\sigma^2_{ax+b} \text{ or } VAR(ax+b) = a^2\sigma^2_x$$

b has no effect at all.

eg Suppose a random variable X has s.d. $\sigma_x = 5$

Calculate:

a) $\sigma^2_{3x} = 9\sigma^2$
$= 9(5)^2$
$= 225$

b) $\sigma^2_{x-3} = \sigma^2_x$
$= 25$

c) $\sigma^2_{2-4x} = \sigma^2_{\cancel{4x}}$
$= 16\sigma^2_x$
$= 16(5)^2$
$= 400$

Joint Distribution

If X & Y are independant random variables, then

$E(X+Y) = E(X) + E(Y)$

σ^2_{X+Y} or $VAR(X+Y) = \sigma^2_x + \sigma^2_y$

$E(X-Y) = E(X) - E(Y)$

σ^2_{X-Y} or $VAR(X-Y) = \sigma^2_x + \sigma^2_x$

It follows from these & previous results that

$E(aX+bY) = a\,E(X) \pm b\,E(Y)$

$\sigma^2_{aX \pm bY}$ or $VAR(aX \pm bY) = a^2\sigma^2_x + b^2\sigma^2_y$

Eg (1) Evan takes his car to the garage for servicin
& a W.O.F.
The time X to service a car has mean 40 min
& s.d. of 8 minutes
The time Y to get a W.O.F. has mean 20 min
& s.d. of 5 minutes.
Find the mean & s.d. of the total time Evans
is required by the garage

$$E(x+y) = E(x) + E(y)$$
$$= 40 + 20$$
$$= 60 \text{ minutes}$$

$$\sigma^2_{x+y} = \sigma_x^2 + \sigma_y^2$$
$$= 8^2 + 5^2$$
$$= 89$$

$$\sigma_{x+y} = \sqrt{89} = 9{\cdot}43 \text{ minutes}$$

(2) Dominic brought his Stats teacher a box containing 100 chocolates.
The mean weight of a box is 20g with a s.d. of 3g.
The mean weight of a chocolate is 3g with s.d. of 0·5g.
Find the mean & s.d. of the weight of a box of chocolates.

We require $E(b+c+c+c+c+c\ldots\ldots)$
$$= E(b) + E(c) + E(c) + E(c) + \ldots\ldots$$
$$= 20 + 100(3)$$
$$= 320g$$

$$\sigma^2_{b+c+c+c+c+c\ldots\ldots} = \sigma_b^2 + \sigma_c^2 + \sigma_c^2 + \sigma_c^2 + \sigma_c^2 \ldots\ldots$$
$$= \sigma_b^2 + 100\,\sigma_c^2$$
$$= (3)^2 + 100(0{\cdot}25)^2$$
$$= 34$$

$$\sigma_{b+c+c+c+c+c\ldots\ldots} = \sqrt{34} = 5{\cdot}8g$$

(3) Bolts are manufactured to fit through wash

The mean diameter of a washer is 18mm wi a s.d. of 3mm

The mean diameter of bolts is 15mm with a s.d. 2mm

Find the mean & s.d. of the clearance when the bo is fitted through the washer

We require $E(w-b)$ & σ^2_{w-b}

$$E(w-b) = E(w) - E(b)$$
$$= 18 - 15$$
$$= 3\text{mm}$$

$$\sigma^2_{w-b} = \sigma^2_w + \sigma^2_b$$
$$= (3)^2 + (2)^2$$
$$= 13$$

$$\therefore \sigma_{w-b} = \sqrt{13} = 3.6\text{mm}$$