Achievement Standard 3.4

Solving Equations

This achievement standard embraces 3 topics:

- simultaneous equations
- linear programming
- iteration methods

Simultaneous Equations

Linear equations in 2 variables represent straight lines.

(1) Solve simultaneously $3x + y = 11$
$x - y = -3$

add $4x = 8$
$x = 2$
$y = 5$

The 2 equations are consistent & have a unique solution. Geometrically, the 2 lines intersect at a unique point $(2, 5)$

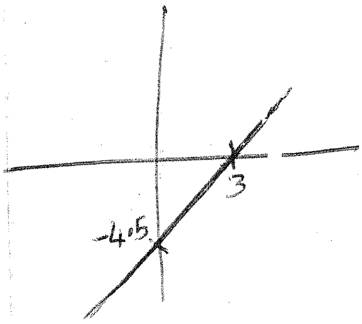
(2) Solve simultaneously

$3x - 2y = 9$ ----- (1)
$6x - 4y = 18$ ----- (2)

$6x - 4y = 18$ ------- (1)×2
$6x - 4y = 18$ ------- (2)

sub $0 = 0$

The two equations are consistent & have an infinite set of solutions.
Geometrically, the 2 lines are the same



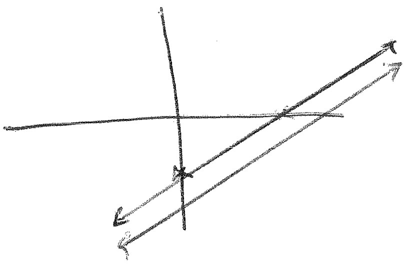
3) Solve simultaneously

$3x - 2y = 14$ ------- (1)
$-6x + 4y = -30$ ------- (2)

$6x - 4y = 28$ ----- (1) $\times 2$
$-6x + 4y = -30$ ----- (2)

add $0 = -2$

The two equations are inconsistent, & no solution exist
Geometrically, these are 2 parallel lines.



3 equations with 3 variables (x, y, z)

The equation $ax + by + cz = d$ represents a plane in 3-D space.
When solving 3 equations in 3 variables the following geometric situations may arise.

(1) The 3 planes meet at a unique point. The system of equations is said to be consistent with a unique solution.

(2) The 3 planes intersect along a line. The system of equations is said to be consistent with an infinite number of solutions

(3) The 3 planes may be parallel OR 2 planes intersect along a line, & the 3rd plane is parallel to this line.
In this case the system of equations is inconsistent & no solution exists.

Examples:

(1) Solve
$$x + 2y + z = 3 \ldots\ldots (1)$$
$$x + y + 2z = 2 \ldots\ldots (2)$$
$$2x + 3y + z = 1 \ldots\ldots (3)$$

eliminate z using (1) & (2)

$$2x + 4y + 2z = 6 \ldots\ldots (1)\times 2$$
$$x + y + 2z = 2 \ldots\ldots (2)$$
sub $x + 3y + 0 = 4 \ldots\ldots (4)$

eliminate z using (1) & (3)

$$x + 2y + z = 3 \ldots\ldots (1)$$
$$2x + 3y + z = 1 \ldots\ldots (2)$$
(2)-(1) = $x + y + 0 = -2 \ldots\ldots (5)$

Now $x + 3y = 4 \ldots\ldots (4)$
$x + y = -2 \ldots\ldots (5)$

subtract $2y = 6$
$y = 3$

Using (5) $x + y = -2$
$x + 3 = -$
$x = -5$

Using (1) $x + 2y + z = 3$
$-5 + 6 + z = 3$
$1 + z = 3$
$z = 2$

Solution is $(x, y, z) = (-5, 3, 2)$

(2) Solve $x+y-z=-2 \ldots (1)$
$5x+2y+z=5 \ldots (2)$
$2x-y+4z=11 \ldots (3)$

Using (1) & (2)

$$\begin{array}{l} x+y-z=-2 \\ \underline{5x+2y+z=5} \\ \text{add } 6x+3y=3 \ldots (4) \\ \quad 2x+y=1 \end{array}$$

Using (1) & (3)

$$\begin{array}{l} 4x+4y-4z=-8 \\ \underline{2x-y+4z=11} \\ 6x+3y+0=3 \ldots (5) \\ \quad 2x+y=1 \end{array}$$

Using (4) & (5)

$$\begin{array}{l} 6x+3y=3 \\ \underline{6x+3y=3} \\ \quad 0=0 \end{array}$$

This always suggests an infinite number of solutions.
(The 3 planes intersect along a line)

Let $x=k$

Using (4) $6k+3y=3$
$3y=3-6k$
$y=1-2k$

Using (2) $5k+2(1-2k)+z=5$
$5k+2-4k+z=5$
$k+2+z=5$
$z=3-k$

Solution is $(x,y,z)=(k, 1-2k, 3-k)$

(3) solve: $x-3y+4z=1 \ldots 1$
$5x-8y+11z=0 \ldots 2$
$2x+y-z=2 \ldots 3$

use (1) & (2)

$$\begin{array}{l} 20x-32y+44z=0 \ldots 2\times 4 \\ \underline{11x-33y+44z=11 \ldots 1\times 11} \\ \text{sub } 9x+y=-11 \end{array}$$

use (1) & (3)

$$\begin{array}{l} x-3y+4z=1 \\ \underline{8x+4y-4z=8} \\ 9x+y=9 \end{array}$$

$$\begin{array}{l} 9x+y=-11 \\ \underline{9x+y=9} \\ \text{sub } \quad 0=-20 \end{array}$$ (an impossible statement)

No solution exists. The system of equations is inconsistent.

Geometrically the 3rd plane is parallel to the line of intersection of the other 2 planes